1. PV of a single cash flow

2. PV of multiple cash flows
   a. Varying
   b. Perpetuity
   c. Growing Perpetuity
   d. Annuity
   e. Growing Annuity

3. PV of a Bond

4. Effective Annual Yield (EAY or EAR) vs. APR

**SAMPLE QUESTIONS**
( Assume the discount rate for all questions is 4% EAY unless it says otherwise)

1. PV of a single cash flow:
   a. Simple
      i. What is the PV of receiving $1000 five years from now?
         
         $1000/(1.04)^5 = 821.9271070$

   b. Long-term half year
      i. What is the PV of receiving $1000 5.5 years from now?
         
         $1000/(1.04)^{5.5} = 805.9658378$

   c. Long term odd length
      i. What is the PV of receiving $1000 five years, one week, and two days from now?
         
         $1000/(1.04)^{(5+9/365)} = 1000/(1.04)^{5.02465753} = 821.1326165$

   d. Short term odd length
      i. What is the PV of receiving $1000 three months from now?
         
         $1000/(1.04)^{0.25} = 990.2427353$
2. **PV of a multiple cash flows:**
   a. **Just two:**
      i. What is the PV of receiving $300 in year one and $250 in year two?
         \[ \frac{300}{1.04} + \frac{250}{1.04^2} = 519.6005918 \]
   b. **Perpetuity**
      i. What is PV of receiving $200 every year for forever starting one year from now?
         \[ \frac{200}{0.04} = 5000 \]
   c. **Growing Perpetuity:**
      i. MAG stock will pay a $2.00 dividend next year. Every year thereafter, it will grow by 2%. What is the most you should pay for MAG?
         \[ \frac{2.00}{0.04 - 0.02} = \frac{2.00}{0.02} = 100 \]
   d. **Annuity:**
      i. You will get paid $300 a year for the next 30 years, starting one year from today. What is the PV?
         \[ 5,187.609990 \]
      ii. You are twenty years old today, and plan on retiring on your 65th birthday. You think you will die on your 95th birthday. You would like to live at $120,000 a year during your retirement. How much money do you need to save each year so that you will just run out of money on your 95th birthday? Assume that you save at the end of the year and all bills are due at the end of the year, so at 95, you pay your bills and then die. (This is somewhat hard to do.)
         First PV 30 years of 120,000 a year at 4%: \( 2,075,043.996 \) needed at 65
         Now PV that back to age 20: \( 2,075,043.996 \) \( \div (1.04)^{45} \) = \( 355,244.2367 \)
         Now find payment on 45 yr annuity so that PV of payment is 355,244.2367: \( 17,144.95927 \)
         So if you put \$17,144.95927 each year for 45 years at 4% you will have \$2,075,043.997 in 45 years at 65 and then at age 66 you can take out \$120,000 a year for the next 30 years and after you take out the \$120,000 on your 95th birthday you will basically have no money left in the account.
e. Growing Annuity:
   i. You will get $500 next year, and you will get 19 more payments. However, each payment will be 3% larger than the previous payment. What is the PV of this stream?

\[
\frac{500}{0.04 - 0.03} \times [1 - (1.03/1.04)^{20}] = 8,785,720.445
\]

3. PV of a Bond:
   a. What is the PV of a five year 8% annual coupon bond with $1000 face value?

\[
$1,178.072893
\]

4. Effective Annual Yield vs. APR?
   a. A bank offers to lend you money at 8%, compounded semi-annually. What is the EAY?

\[
(1.04)^2 = 1 + \text{EAY} = 1.0816 \quad \text{so EAY}=8.16\%
\]

b. Now they offer to lend you money at 7.8%, compounded quarterly instead of the offer above. You are going to borrow money for a year. Which should you take?

\[
7.8%/4 = 1.95\% \text{ every three months.}
\]
\[
(1.0195)^4 = 1 + \text{EAY} = 1.08031130 \quad \text{so EAY} = 8.031130\%
\]

So take this one. It is a little cheaper.

c. What is the price of a seven year 10% semi-annual coupon bond if the discount rate is 12% compounded monthly?

Assume Face value is $1000. Then coupon is $50 every six months.

The discount rate is 12%/12, or 1% per month.

We need the effective rate for six months. So, \((1.01)^6 = 1.061520151\)

Six month effective rate is 6.1520151%

(Note, you can also get this by getting EAR: \((1.01)^{12} = 1.12682503\)
And then take the square root)

So,
N=14
PMT = 50
FV = 1000
I= 6.1520151

So:
\[
\text{PV} = 893.9211426
\]