

6 Appendix

6.1 Derivations for Section 4.2

To clarify the exposition in section 4.2, we omitted the details regarding the market maker's updating rules. For completeness, we present those updating rules here. We adopt the notation:

$$\rho_N = \Pr\{X_2^S = 0, X_2^L = 0\} = \text{probability informed agent defers from trade at period 2;}$$

$$\rho_L = \Pr\{X_2^S = 0, X_2^L = 1\} = \text{probability informed agent buys a long-dated contract at period 2;}$$

$$\rho_S = \Pr\{X_2^S = 1, X_2^L = 0\} = \text{probability informed agent buys a short-dated contract at period 2.}$$

If the market maker observes three (or more) orders in aggregate ($Y_2^S + Y_2^L \geq 3$) in period 2, she knows the good state occurred and updates her beliefs accordingly ($\beta_2 = 1$). The remainder of the period 2 market maker updating rules are:

$$\begin{aligned} \beta_2(0, 0) &= \frac{1 - \delta[\rho_L + \rho_S]}{2 - \delta[\rho_L + \rho_S]} \\ \beta_2(1, 0) &= \frac{(3 - 2\xi)(1 - \delta + \delta\rho_N) + 2\delta\rho_S}{(3 - 2\xi)(2 - \delta + \delta\rho_N) + 2\delta\rho_S} \\ \beta_2(0, 1) &= \frac{(1 + 2\xi)(1 - \delta + \delta\rho_N) + 2\delta\rho_L}{(1 + 2\xi)(2 - \delta + \delta\rho_N) + 2\delta\rho_L} \\ \beta_2(1, 1) &= \frac{1 + 2\delta(\xi\rho_S + (1 - \xi)\rho_L)}{2 + 2\delta(\xi\rho_S + (1 - \xi)\rho_L)} \\ \beta_2(2, 0) &= \frac{(3 - 2\xi)\delta\rho_S + (1 - \xi)(1 - \delta + \delta\rho_N)}{(3 - 2\xi)\delta\rho_S + (1 - \xi)(2 - \delta + \delta\rho_N)} \\ \beta_2(0, 2) &= \frac{2\xi\delta\rho_L + \xi\delta\rho_N + \xi(1 - \delta) + \delta\rho_L}{2\xi\delta\rho_L + \xi\delta\rho_N + \xi(2 - \delta) + \delta\rho_L} \end{aligned}$$

The market maker's period 1 updating rules reflect the fact that period 2 order flow provides information about the existence of the long-lived liquidity trader:

$$\begin{aligned} \beta_1(0, (\cdot, \cdot)) &= \frac{1 - \delta}{2 - \delta} \\ \beta_1(1, (0, 0)) &= \frac{1 - \delta[\rho_L + \rho_S]}{2 - \delta[\rho_L + \rho_S]} \end{aligned}$$

$$\begin{aligned}
\beta_1(1, (0, 1)) &= \frac{1 + 2\xi - \delta[(1 + 2\xi)(1 - \rho_N) - 2\rho_L]}{2 + 2\xi - \delta[(1 + 2\xi)(1 - \rho_N) - 2\rho_L]} \\
\beta_1(1, (1, 0)) &= \frac{(3 - 2\xi)(1 - \delta) + 2\delta\rho_S + \delta\rho_N}{(3 - 2\xi)(2 - \delta) + 2\delta\rho_S + \delta\rho_N} \\
\beta_1(1, (1, 1)) &= \frac{1 - \delta[(1 + \xi)\rho_N - 2\xi\rho_S]}{2 - \delta[(1 - \xi)\rho_N - 2\xi\rho_S]} \\
\beta_1(1, (0, 2)) &= \frac{2\xi\delta\rho_L + \xi\delta\rho_N + \xi(1 - \delta) + \delta\rho_L}{2\xi\delta\rho_L + \xi\delta\rho_N + \xi(2 - \delta) + \delta\rho_L} \\
\beta_1(1, (2, 0)) &= \frac{(1 - \xi)(1 - \delta) + \delta\rho_S}{(1 - \xi)(2 - \delta) + \delta\rho_S} \\
\beta_1(2, (1, 1)) &= \frac{2\xi\delta\rho_S + 3\delta\rho_L - 2\xi\delta\rho_L + \delta\rho_N + (1 - \xi)(1 - \delta) + \delta\rho_S}{2\xi\delta\rho_S + 3\delta\rho_L - 2\xi\delta\rho_L + \delta\rho_N + (1 - \xi)(2 - \delta) + \delta\rho_S} \\
\beta_1(2, (2, 0)) &= \frac{2(1 - \xi)\delta\rho_S + (1 - \xi)\delta\rho_N + \delta\rho_S + (1 - \xi)(1 - \delta)}{2(1 - \xi)\delta\rho_S + (1 - \xi)\delta\rho_N + \delta\rho_S + (1 - \xi)(2 - \delta)} \\
\beta_1(2, (1, 0)) &= \frac{(1 - \xi)\delta\rho_N + (1 - \xi)(1 - \delta) + \delta\rho_S + 0.5\delta\rho_N}{(1 - \xi)\delta\rho_N + (1 - \xi)(2 - \delta) + \delta\rho_S + 0.5\delta\rho_N}
\end{aligned}$$

If the market maker received three or more orders in aggregate in either period 1 or 2, then she assigns $\beta_1 = 1$. There are three additional order flow combinations that reveal the existence of the informed agent to the market maker:

$$\beta_1(2, (0, 0)) = \beta_1(2, (0, 1)) = \beta_1(2, (0, 2)) = 1$$

Endogenous liquidity trader's expected cost from buying a long-dated contract at period 2:

$$\begin{aligned}
E[C^L] &= 0.25 \left[\delta(1 - \rho_N) + (1 + 0.5\delta\rho_N - 0.5\delta)(\beta_2(0, 2) + \beta_2(1, 1) + 2\beta_2(0, 1)) \right. \\
&\quad \left. + \delta\rho_L\beta_2(0, 2) + \delta\rho_S\beta_2(1, 1) \right] + c
\end{aligned}$$

Endogenous liquidity trader's expected cost from buying a short-dated contract each period:

$$\begin{aligned}
E[C^S] &= 0.25 \left[\delta(1 - \rho_N) + (1 + 0.5\delta\rho_N - 0.5\delta + \delta\rho_L)\beta_2(1, 1)\lambda_2 + (1 + 0.5\delta\rho_N - 0.5\delta \right. \\
&\quad \left. + \delta\rho_S)\beta_2(2, 0)\lambda_2 + 2(1 + 0.5\delta\rho_N - 0.5\delta)\beta_2(1, 0)\lambda_2 \right] + c + 0.125(1 - \lambda_2) \left[0.5\delta(\rho_N + 2\rho_L) \right. \\
&\quad \left. + (0.5\delta\rho_N + 1 - 0.5\delta + \delta\rho_L)\beta_1(2, (1, 1)) + (1 - 0.5\delta)\beta_1(1, (1, 1)) + 0.5\delta(\rho_N + 2\rho_S) \right. \\
&\quad \left. + (0.5\delta\rho_N + 1 - 0.5\delta + \delta\rho_S)\beta_1(2, (2, 0)) + (1 - 0.5\delta)\beta_1(1(2, 0)) + \delta\rho_N \right. \\
&\quad \left. + 2(1 - 0.5\delta + 0.5\delta\rho_N)\beta_1(2(1, 0)) + 2(1 - 0.5\delta)\beta_1(1(1, 0)) \right] + (1 - \lambda_2)(1 - 0.25\delta(\rho_S + \rho_L))c
\end{aligned}$$

Endogenous liquidity trader's expected cost of hedging: $E[C^T] = \xi E[C^L] + (1 - \xi)E[C^S]$.