Long-term Information, Short-lived Securities*

Dan Bernhardt[a], Ryan J. Davies[b,†], John Spicer[c]

[a]*IBE Distinguished Professor in the Department of Economics at the University of Illinois in Champaign, Illinois.*

[b]*Assistant Professor in the Finance Division at Babson College in Babson Park, Massachusetts.*

[c]*Senior Consultant at Analysys in London, England.*

August 8, 2005

*The views expressed are those of the authors. We are grateful for comments received from David Brown, Melanie Cao, Michael Goldstein, Laurie Krigman, Bob Webb (the editor), an anonymous referee, and seminar participants at: Queen's University, University of Toronto, Dalhousie University, Simon Fraser University, Memorial University, Western Michigan University, University of Miami, Fordham University, University of Colorado, Federal Reserve Bank of New York; the Western Finance Association Meetings; the Canadian Economic Theory Meetings; and the Northern Finance Association Meetings.

†Corresponding author. Finance Division, Babson College, Babson Park, MA 02457–0310, USA. Tel: +1–781–239–5345; fax: +1–781–239–5004. *E-mail address:* rdavies@babson.edu (R.J. Davies).

Long-term Information, Short-lived Securities

Abstract

This paper explores strategic trade in short-lived securities by agents who have private information that is potentially long-term, but do not know how long their information will remain private. Trading short-lived securities is profitable only if enough of the private information becomes public prior to contract expiration; otherwise the security will worthlessly expire. We highlight how this results in trading behavior fundamentally different from that observed in standard models of informed trading in equity. Specifically, we show that informed speculators are more reluctant to trade shorter-term securities too far in advance of when their information will necessarily be made public, and that existing positions in a shorter-term security make future purchases more attractive. Because informed speculators prefer longer-term securities, this can make trading shorter-term contracts more attractive for liquidity traders. We characterize the conditions under which liquidity traders choose to incur extra costs to roll over short-term positions rather than trade in distant contracts, providing an explanation for why most longer-term derivative security markets have little liquidity and large bid-ask spreads.

JEL Classification: G1; D82

Keywords: Private information; Derivative securities; Rolling the hedge; Fixed transaction costs

1 Introduction

This paper models strategic trade in *short-lived* securities by an "informed agent" or "speculator" who has private information that is potentially long-term, but who is uncertain about how long he will retain his informational advantage before his information is publicly revealed to the market. In the first part of the paper, we consider an informed agent who has the opportunity to trade in a series of short-lived securities: as time passes, newly-issued contracts with later expiration dates become available for purchase. From the speculator's perspective, trade in a short-lived security is profitable only if enough of the private information becomes public (and is reflected in prices) prior to contract expiration; otherwise the security will worthlessly expire. Further, the speculator's decision about whether or not to enter the market this period has implications for the prices of contracts in future periods. Thus, the informed agent must decide when, and how frequently, to purchase these contracts. This paper is the first to demonstrate how the short-lived nature of these contracts leads to different dynamic trading outcomes than standard models of speculative trade in equity.

In the second part of the paper, we extend the basic model to consider a speculator who can choose between trading short- and long- dated contracts. This lets us provide insights into a puzzling feature of many derivative security markets: despite the existence of a widespread desire to hedge against long-term risk, as time-to-maturity increases, longer-term option and futures contracts have less trade volume, lower open interest, and large bid-ask spreads.[1] In fact, heuristic industry evidence suggests that a common strategy for hedging long-dated obligations is to sequentially roll over nearby futures contracts. *Rolling the hedge* has an additional cost over buying longer-term contracts — each time a position is rolled over, transactions costs (commissions, bid-ask spread, basis risk) must be incurred. Why incur these extra costs? Why don't hedgers just use contracts that match their horizon?

We argue that the key to understanding this empirical puzzle is to look at the behavior of liquidity traders (hedgers) and speculators jointly. To investigate why it might be optimal to incur these additional roll-over costs, our extended model introduces an endogenous liquidity trader who can

[1] For example, Fleming and Sarkar (1999) found that 90% of total trading volume in Treasury Futures in 1993 occurred in the *nearby* contract (the contract closest to expiration), and that distant contracts generally had far *lower trading volumes* and *larger realized spreads*.

either (i) buy the short-dated contract and roll it over or (ii) buy the long-dated contract. We then show that in reasonable settings hedgers choose to incur extra costs to roll over their short-term positions, rather than trade distant contracts. The intuition is that speculators place a greater value on distant contracts because their information is more likely to be revealed before the contract expires, while hedgers value only the reduced roll-over costs. As long as fixed trading costs are not too large, the greater adverse selection costs in distant contracts more than offset the increased fixed trading costs, so that long-term hedgers prefer to trade nearby contracts. If all hedgers prefer the nearby contract, speculators must also trade it. Thus, a pooling equilibrium results in which virtually all trade occurs in the short-dated contracts.

To fix ideas, it is easiest to think of our model in the context of trading in commodity futures contracts, where a speculator cannot readily take a position by purchasing the commodity (e.g. soybeans); but instead must choose between trading two relatively short-term futures contracts (say, a 2-month contract and a 4-month contract). Later, we discuss how our model's findings extend to more general environments, including when the underlying is storable or when comparing liquidity between nearby contracts and very long-dated contracts deliverable in several years.

Our characterizations reveal that for reasonable levels of fixed trading costs, long-dated contract markets can thrive only in environments where informational asymmetries are slight. In fact, unless long-term private information is both extremely rare and unlikely to be revealed in the short-term, most long-dated obligations are hedged with short-dated contracts. Thus, markets with minimal informational asymmetries (e.g., weather derivatives) might be expected to have liquid markets at longer horizons; but markets with substantial informational asymmetries (e.g., equity derivatives) should not. That said, we suggest a reason why exchanges might still have the incentive to incur high costs to introduce long-dated contracts: to the limited extent that long-dated contracts attract informed trade, they reduce adverse selection costs in short-dated contracts, thereby improving the prices of the short-dated contracts, and reducing the total cost of hedging long-term obligations.

Our basic model shares features of the overlapping generations model proposed by Dow and Gorton (1994). Dow and Gorton endogenize the choice to act on private information made by speculators with short-horizons and *one* trading opportunity. In their model, information is only revealed through trade. Hence, a speculator only trades if he believes it sufficiently likely that a future speculator will trade before he has to realize his position, so that prices will reflect his information. Because Dow and Gorton allow a speculator only one chance to trade, their model does not capture the essence of repeatedly trading short-lived securities when speculators have long-term intrinsic private information. In particular, their speculators do not internalize the effects of current trade on future trading opportunities, so that immediate trade is more attractive and private information becomes public sooner.

In contrast to the Dow and Gorton model, we consider a *single* informed agent contemplating purchases of a *series* of short-lived securities. We show that the speculator's trading decisions and profit depend subtly on both the likelihood of liquidity trade and how far the market maker's competitive price is expected to diverge from its 'fundamental' value. We then extend the model to consider how the speculator's strategy depends on his accumulated position and the availability of contracts of different durations.

Each time a speculator purchases a short-term security he conveys some of his private information to the market, *even* if the contract does not pay off. The information impact of the purchase adversely affects both current and future contract prices. Thus, the strategic information costs of trading are higher for short-term securities than for equity, and more generally for nearby contracts than distant contracts. This is because, in contrast to trading equity, where an informed agent will ultimately benefit from his trades, trading short-term securities is profitable only if the private information is impounded in the price before expiry. Ignoring leverage concerns, this makes a speculator more reluctant to trade immediately in shorter-term securities. Indeed, even if it is profitable in expectation to trade now, an informed agent may prefer to defer in the hope of obtaining better prices in the future. The attendant risk with this strategy is that his information may be revealed publicly before he can exploit it.

Thus, our model contrasts with models of informed trading in equity (e.g., Kyle, 1985; Holden & Subrahmanyam, 1992; Foster & Viswanathan, 1996; Back, Cao, & Willard, 2000) where speculators have multiple opportunities to trade a long-lived asset, but do not need to realize positions before their private information becomes public. These speculators care about both current and future prices, but because they do not have to unwind positions there is no penalty to investing early; eventually stock prices reflect private information.

We show that an informed agent with a larger accumulated position of short-lived securities trades more aggressively: submitting an order raises the probability that his information is revealed and that his previously purchased contracts will then be exercised at a profit. Thus, in further contrast to equity, a risk-neutral speculator's holding of a short-lived security affects his trading behavior: higher past informed trading leads to greater future informed trading. The higher are a speculator's holdings of the short-lived security, the less he minds if his subsequent trading reveals his information, as he will profit from his existing stake. In contrast, equity holdings do not affect his trading strategy in the same manner, as he will eventually profit on his existing stake, as long as he does not close his position. So, too, the time-to-expiry of the short-lived affects trading intensities: speculative trading intensities rise as expiry approaches, thereby increasing price sensitivity to order flow.

To date, the academic literature has largely ignored these important issues. In a related paper, Back (1993) integrates a long-lived call option into a continuous time Kyle (1985) model. Back shows that if trades in the option and the underlying asset convey different information, then the option cannot be spanned by the underlying asset (i.e. it is not redundant) and thus cannot be priced by arbitrage arguments. Indeed, the empirical observations that long-dated contracts typically have little volume and large bid-ask spreads can only be reconciled by the presence of market microstructure effects.

Unfortunately, one cannot modify Back's model of a long-lived option to explain these empirical regularities and capture the strategic impacts of trading short-lived securities. Incorporating short-lived securities immediately entails consideration of how asset holdings affect strategic trading behavior. Also, much of the strategic tradeoff between long- and short-lived securities for both liquidity and informed agents concerns the avoidance of fixed trading costs associated with rolling over positions. The requirement that trade be normally distributed precludes the possibility that a Kyle-style model such as Back's can capture the strategic tradeoff between higher fixed trading costs of rolling over shorter-term securities, and their reduced adverse selection costs.

Biais and Hillion (1994) and John, Koticha, Narayanan, and Subrahmanyam (2001) also model informed trade in option markets. Biais and Hillion consider a static model in which a single trader (either informed or a rational, risk averse liquidity trader) chooses whether to trade the stock or the option. Adding the opportunity to trade options may reduce informed profits because of the effects on liquidity trading strategies. John et al.'s static model explores how margin requirements affect the choice by informed agents of which asset to trade. Although related to ours, these static models cannot shed light on the dynamic trading issues that we address.

Past research on hedging strategies has focused on finding the optimal roll-over strategy rather than on answering why the relevant long-dated contract has such poor liquidity.[2] One strand of literature that does touch on time-to-maturity issues is that of the so-called Samuelson effect. Samuelson (1965, 1976) shows that under certain assumptions about price stationarity, near futures contracts show more variability than (sufficiently far) distant ones. To the extent that greater contemporaneous variability leads to greater trade, this effect has been proposed as a possible explanation for why nearby contracts have greater liquidity than distant contracts.

The empirical evidence of the Samuelson effect, however, is mixed.[3] And, the Samuelson effect is derived based on the time between expiration of the nearby and distant contract becoming asymptotically large — this asymptotic result cannot be used to reconcile why there is a large drop-off in volume between, say, 2-month and 4-month futures contracts. Nor does it, in itself, explain why bid-ask spreads are so much larger, or open interest is so much smaller, in long-dated contracts. While greater variability in short-dated contracts may benefit speculators, hedgers would actually benefit from trading longer dated contracts because their hedges would have to be reset less often.

[2]See, for example, Brennan and Crew (1997), Gardner (1989), Lence and Hayenga (2001), Lien and Wang (2004), McCabe and Franckle (1983), Veld-Merkoulova and De Roon (2003). Agents must pay for the "spread" between two futures contracts with different expiration dates. This cost may be significant if there are systematic differences in the basis between the local cash price and the price of futures contracts with different durations. For an analysis of the implications of rollover risk, see Neuberger (1999), who derives optimal long-term hedging strategies using multiple short-term futures contracts to minimize this risk.

[3]For instance, Castelino and Francis (1982) and Movassagh and Modjtahedi (2005) find empirical support of the Samuelson effect in a variety of commodity futures contracts; whereas Rutledge (1976) and Leistikow (1989) find the effect occurs in some commodity markets but not others. The evidence is far weaker in financial futures (e.g., Han, Kling, & Sell, 1999). Some support for the effect has been found in energy futures (Herbert, 1995; Walls, 1999).

The Samuelson effect does not hold in environments with information asymmetry, such as we model here. For example, Hong (2000) shows that varying the degree of information asymmetry among investors can lead to violations of the Samuelson effect and to different time-to-maturity patterns of open interest.[4] Our analysis differs from his in that he compares outcomes of each different time-to-maturity length independently, whereas we examine the equilibrium outcomes of simultaneously selecting between trade in two contracts of different time-to-maturities.

The paper is organized as follows. We next set out the basic model. Section 3 characterizes the informed agent's equilibrium trading strategy and details how outcomes vary with the parameters describing the economy. Section 4 introduces short-lived contracts that exist for longer periods, allowing us to consider the impact of the speculator's accumulated position and to consider liquidity in long- and short-lived securities. Section 5 concludes. Proofs are in an appendix.

2 The Basic Model

Consider a multi-period economy with two primitive assets: a riskless asset that returns $r = 0$, and a risky asset. At some distant date T, the value of the risky asset will be $v \in \{B, G\}$, $G > B$. *Ex ante* the good state ($v = G$) and the bad state ($v = B$) occur with equal probability. Let $i = T - t$ be the number of periods *remaining* until date T: Period i corresponds to the date with i periods remaining until date T.

There are three types of market participants: (1) a risk neutral, competitive market maker; (2) short-lived liquidity traders who arrive randomly; and (3) an informed agent. With probability δ, a speculator has private information about the risky asset: he knows whether the realization was G or B. The true future state may be revealed to the public in period i in two ways: either by the equilibrium trading order flow (in a manner detailed below); or by an information leak, which occurs with probability λ_i.[5] As period 0 approaches, the asset's value is more likely to be revealed

[4]Routledge, Seppi, and Spatt (2000) also show that conditional violations of the Samuelson effect can occur in the case of a storable commodity. This occurs because the spot commodity has an embedded timing option that is absent in forward contracts. Anderson and Danthine (1983) show that futures price volatility may increase or decrease as delivery approaches depending on the pattern of information flow into the market.

[5]An alternative interpretation of our economic environment is that the risky asset pays a dividend of B or G at a

to the public, $\lambda_i \leq \lambda_{i-1}$. Let $\theta_i \in \{b, n, g\}$ reflect whether bad news (b), no news (n), or good news (g) leaked out at period i.

Market participants can trade a series of one-period European binary call options. The option is available for purchase at the beginning of each period i at price $P_{o,i}$ and expires at the end of period i for a payoff of $P_{c,i}$. At the beginning of period i, given current and past order flows, the competitive market maker assigns a probability $\beta_{o,i}$ to the good state. She then prices the option sold at the beginning of period i at

$$P_{o,i} = \begin{cases} 1 & \text{if } \beta_{o,i} = 1 \\ \beta_{o,i}\lambda_i & \text{if } \beta_{o,i} < 1. \end{cases} \tag{1}$$

At the end of period i, the market maker updates her belief about the good state to $\beta_{c,i}$ to reflect the possible leakage of private information during the period. The payoff at expiration to an option that matures at the end of period i is:

$$P_{c,i} = \begin{cases} 1 & \text{if } \theta_i = g \\ 0 & \text{if } \theta_i = n \text{ or } \theta_i = b. \end{cases}$$

We focus on binary options for the same reason that Dow and Gorton (1994) consider assets that pay off 0 or 1 — they reduce algebra and the qualitative predictions extend to more general short-lived contracts (e.g. standard options, futures contracts). Specifically, the binary options are in the money if and only if the private information is revealed before they expire. In this way we capture the essence of trading short-lived securities written on longer-lived assets, while circumventing the need to model trade in multiple markets at each date. Section 4 extends the model to allow for multiple securities of different durations, and the choice of which security to trade.

All agents have a sufficient endowment that they are not wealth constrained in equilibrium. Agents incur a transaction cost of $c \geq 0$ when buying or selling the short-term security. This fixed trading cost incorporates all brokerage fees, time costs, and the expected cost of an unmodeled rollover risk.[6] Without loss of generality, we assume that agents can only submit orders in round (integer) lots (this assumption never affects equilibrium outcomes).

random time between dates $t = 0$ and $t = T$. The speculator knows the dividend realization but not the timing of the dividend payment. Then λ_i corresponds to the probability the dividend is paid at period i, conditional on the dividend not previously being paid.

[6]Our model has no explicit rollover risk because the basis between the cash price and futures contract price is always zero, for all maturities. Effectively, in our model, the expected cost of rollover risk is included in the fixed

The basic model only considers the "buy" side of the market.[7] As a result, the subgame equilibrium when the bad state occurs just has the speculator declining to place an order. Since we want to consider situations where the speculator may trade, without loss of generality, we focus on the case where the good state occurs.

It is important to emphasize that we can extend our model to incorporate the "sell" side of the market. This has a non-trivial effect on outcomes, but *not* on any of our qualitative findings. If market participants could trade on both sides of the market, then for some parameterizations the speculator might want to (probabilistically) trade against his information in order to manipulate market maker beliefs. In equilibrium, however, the market maker accounts for this possibility, revising her beliefs less dramatically in response to observed order flow.[8] Allowing for this possibility does not otherwise change the model's qualitative predictions, but it does complicate speculator strategies and reduce the clarity with which private information is revealed through trade. It is for these reasons that we focus on the "buy" side.

Short-lived liquidity traders: Each period i, a short-lived liquidity trader enters the market with probability $\frac{1}{2}$ to place a buy order of size one for the short-term security. The liquidity trader is uninformed and trades only once. With equal probability there is no liquidity trade in period i. Let $Z_i \in \{0, 1\}$ represent the liquidity trader's period i order. This stark contrast between high noise trading (one order) and low (no orders) allows the model to capture simply the feature that the market maker detects a speculator's presence with positive probability.

Informed speculator: There is at most one privately-informed speculator in the market. The speculator can trade the short-term security as often as he wishes. Of course, given positive transactions costs, he will not trade once his information becomes public. The speculator and the possible liquidity trader submit their orders simultaneously. A risk neutral competitive market maker observes the aggregate order flow and sets a price equal to the conditional expected value of the short-term

trading cost. We do not model this added rollover cost explicitly because the risk is largely orthogonal to the adverse selection concerns that are the focus of our analysis. Incorporating rollover risk into the fixed costs simplifies the model, making it easier to identify the impact of adverse selection.

[7] Our modeling assumptions can be motivated by Easley, O'Hara, and Srinivas (1998). They document empirically that (i) option markets are a key venue for information-based trading; and (ii) it is important to distinguish between 'positive' and 'negative' news option volumes.

[8] Goldstein and Guembel (2004) illustrate how manipulation can be incorporated into a similar model.

security given her information.

The sequence of events during period i is as follows:

1. The market maker enters with a prior $\beta_{c,i+1}$ that reflects past trade and announcements.
2. The speculator, having observed past order flows and public announcements, selects a trading probability. With probability $\frac{1}{2}$, a liquidity trader also submits an order.
3. The market maker observes total order flow, updates her beliefs to $\beta_{o,i}$, and sets an opening option price of $P_{o,i}$.
4. During the period, the speculator's private information may or may not be revealed. This information is revealed publicly with probability λ_i.
5. At period's end, the market maker updates her prior ($\beta_{c,i}$) to reflect whether the private information was revealed publicly.
6. If the good state was revealed, agents can exercise the options (and receive 1 for each option) or sell them at their closing price, $P_{c,i} = 1$. If the good state was not revealed, the options expire worthlessly, $P_{c,i} = 0$.

Equilibrium: The total period i order flow for the short-lived security is $Y_i = X_i + Z_i$. Let $\mathbf{H}_i = \{Y_T, Y_{T-1}, ..., Y_i\}$ and $\mathbf{\Theta}_i = \{\theta_T, \theta_{T-1}, ..., \theta_i\}$ denote respectively, the period i history of past order flows and past public announcements. The order submission function, Pr $\{X_i = x_i \mid v, \mathbf{H}_{i+1}, \mathbf{\Theta}_{i+1}\}$, is a period strategy for the speculator, mapping the date T asset value and history of order flows and announcements into a probability distribution over the set of feasible individual orders for the short-lived security for each period i. A period strategy for the market maker is a pair of pricing functions, $P_{o,i}(\mathbf{H}_i, \mathbf{\Theta}_{i+1})$ and $P_{c,i}(\mathbf{H}_i, \mathbf{\Theta}_i)$, for the open and close of trading respectively, that map the order flow and public announcement histories into prices.

In a sequentially rational (perfect Bayesian) equilibrium: (i) the speculator's order submission strategy maximizes (recursively) lifetime expected profits given correct beliefs about pricing functions; and (ii) the pricing function is consistent with the speculator's behavior and earns the market maker zero expected profits conditional on the order flow.

We solve for the equilibrium recursively. If the total order flow in the market exceeds one or is a non-integer quantity, the market maker knows that the speculator traded. Hence, if the speculator submits an order, he will try to conceal his trade from the market maker by placing an order for one round lot: in equilibrium, $X_i \in \{0, 1\}$. For simplicity, let $\chi_i = \Pr\{X_i = 1\}$ denote the probability that the speculator submits an order for one round lot at period i; $1 - \chi_i$ is the probability that the speculator defers from trading.

Given competitive pricing, in equilibrium, the history of order flow and public announcements through period i can be summarized by the market maker's belief at the end of period i that the good state will occur, $\beta_{c,i}$. Since the speculator only trades when $v = G$, the speculator's strategy can be summarized by $\chi_i(\beta_{c,i+1})$. In equilibrium, the market maker's pricing strategy at period i can also be summarized by $P_{o,i}(\beta_{c,i+1}, Y_i)$ and $P_{c,i}(\beta_{c,i+1}, Y_i, \theta_i)$.

We now develop the economy formally. We first derive the beliefs the market maker forms about the probability that the good state will occur. At the beginning of period T, market maker beliefs correspond to the *ex ante* probability that the good state occurs: $\beta_{c,T+1} = \frac{1}{2}$. At any period $i \leq T$, the market maker will receive either zero, one or two orders as illustrated in figure 1. An application of Bayes' rule shows that the market maker assigns equilibrium probability

$$\beta_{o,i}(\beta_{c,i+1}, Y_i) = \begin{cases} \frac{[1-\delta\chi_i(\beta_{c,i+1})]\beta_{c,i+1}}{1-\delta\chi_i(\beta_{c,i+1})\beta_{c,i+1}} & \text{if } Y_i = 0 \\ \beta_{c,i+1} & \text{if } Y_i = 1 \\ 1 & \text{if } Y_i = 2 \end{cases} \tag{2}$$

to the good state.[9] These updating rules reflect the *equilibrium* outcome resulting from the consistency of market maker beliefs and the speculator's actions. The probability $\beta_{o,i}(\beta_{c,i+1}, Y_i)$ determines the opening price at which the contract is purchased in period i. The closing price reflects whether the true state was revealed publicly. If the information is not revealed, $\beta_{c,i} = \beta_{o,i}$. Otherwise, $\beta_{c,i} = 1$ if the good state is revealed and $\beta_{c,i} = 0$ if the bad state is revealed. To reduce notation, in what follows we denote $\beta_{c,i}$ as simply β_i.

[9]For example, this reflects that there are three possible situations in which no orders will be submitted: (a) with probability, $\frac{\delta}{2}(1 - \beta_{c,i+1})$, no liquidity trader enters and the speculator sees bad news; (b) with probability, $\frac{\delta}{2}\beta_{c,i+1}[1 - \chi_i(\beta_{c,i+1})]$, no liquidity trader enters and the speculator receives good news but refrains from trading; and (c) with probability $\frac{(1-\delta)}{2}$, no liquidity trader enters and there is no speculator.

3 Speculator's Optimization Problem

By solving the informed speculator's dynamic programming problem that determines his trading decisions, we can characterize equilibrium outcomes when we substitute in consistent beliefs of the market maker. The analysis exploits the fact that prices are higher when the market maker believes the speculator is *more* likely to trade, making trading *less* attractive. The functional equation governing the speculator's expected trading profits as a function of market maker beliefs is

$$V_i(\beta_{i+1}) = \max_{X_i \in \{0,1\}} E\Big[\pi_i(\beta_{i+1}, Y_i) + V_{i-1}(\beta_i(\beta_{i+1}, Y_i))\Big], \qquad \text{s.t. } Y_i = X_i + Z_i,$$

where π_i are period payoffs and $V_0(\cdot) = 0$. In equilibrium, the speculator's mixed trading strategy χ_i corresponds to the probability that the market maker assigns to a speculator with good news trading. If equilibrium is characterized by mixing, we can solve for the equilibrium value of χ_i by finding the market maker's belief about the trading probability implicit in the updating rule $\beta_i(\beta_{i+1}, Y_i)$ that leaves the speculator indifferent between trading and not.

The value of the speculator's private information with i periods remaining until his private information is sure to be revealed publicly is:

$$V_i(\beta_{i+1}) = \frac{(1-\lambda_i)}{2} V_{i-1}(\beta_{i+1}) + \frac{1}{2}\max\Big[\lambda_i(1-\beta_{i+1}) - 2c, (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0))\Big], \quad (3)$$

where $V_0 = 0$. In equilibrium, the market maker's beliefs must be consistent with the speculator's period trading strategies. This may require that the speculator use a mixed trading strategy. This reflects the fact that $\beta_i(\beta_{i+1}, Y_i = 0)$ decreases in χ_i. As a result, the market maker's belief about the probability of the good state falls more in response to an order flow of zero when the speculator has a pure strategy to buy than when he has a pure strategy to defer. The equilibrium is characterized by a non-degenerate mixed trading strategy when:

$$(1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0, \chi_i = 1)) > \lambda_i(1-\beta_{i+1}) - 2c > (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0, \chi_i = 0)).$$

If the speculator mixes in equilibrium, he must be indifferent between trading and not:

$$\lambda_i(1-\beta_{i+1}) - 2c = (1-\lambda_i)V_{i-1}(\beta_i(\beta_{i+1}, Y_i = 0)). \quad (4)$$

In period 1, the right-hand side of (4) is zero and the value function is independent of χ_1. Hence, it is easy to compute the speculator's equilibrium trading probabilities:$\chi_1(\beta_2) = 0$ if $\beta_2 > 1 - \frac{2c}{\lambda_1}$, and $\chi_1(\beta_2) = 1$ if $\beta_2 < 1 - \frac{2c}{\lambda_1}$. We now show that in period 2, the more likely the market maker believes the good state, the less likely the speculator is to trade on his (good) private information:

Lemma 1 *The probability that a speculator with good news trades with two periods remaining, $\chi_2(\beta_3)$, decreases in β_3; strictly decreasing with β_3 if it is interior, $0 < \chi_2(\beta_3) < 1$.*

Before extending lemma 1 to the general period i case, it is useful to characterize how informed trading intensities vary as time passes. One might conjecture that the speculator would trade more aggressively when there are fewer remaining trading opportunities. The following example illustrates that the analysis is more subtle. Suppose that $\bar{\beta} = 0.5$, $\delta = 0.4$, $c = 0.1$, $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, and $\lambda_3 = 0.49$. Then, $\chi_3(\bar{\beta}) = 0.897 > 0 = \chi_2(\bar{\beta})$; trading intensities do not rise uniformly as the end of the trading horizon nears. Intuitively, because information is far more likely to become public in period 1 than period 2, (*i.e.*, $\lambda_1 >> \lambda_2$), the gain from deferring in period 2 in hopes of profiting on trade in period 1 is high. Information, however, is only marginally more likely to be revealed at the end of period 2 than period 3, so in period 3 the choice boils down to a choice between trading in period 3 or waiting until period 1. But since his information is sufficiently likely to become public before period 1, it does not pay to defer in period 3.

This non-monotonicity appears to arise because the probability of public information revelation evolves over time in a convex pattern. Indeed, if it does not vary over time, then the speculator's trading intensity must rise as date T is approached:

Proposition 1 *Let $\lambda_i = \lambda,\ \forall i$. Then the speculator trades with positive probability if and only if the price is not too close to its fundamental value, i.e., $\bar{\beta} < 1 - \frac{2c}{\lambda}$. For $\bar{\beta} < 1 - \frac{2c}{\lambda}$, as date T approaches, the speculator is more likely to submit an order: $\chi_i(\bar{\beta}) \geq \chi_{i+1}(\bar{\beta})$, and $\chi_i(\bar{\beta}) > \chi_{i+1}(\bar{\beta})$ if $\chi_{i+1}(\bar{\beta}) < 1$. Hence, as date T approaches, prices become more sensitive to order flow.*

Underlying this result is Lemma 2 (in the appendix), which details that if the probability of information leakage does not vary over time, then the speculator' expected profits rise with the number of periods available to trade on his information: $V_i(\beta) \leq V_{i-1}(\beta)$. Thus, Proposition 1 can be interpreted as follows. When $\lambda_i = \lambda,\ \forall i$, the expected return from submitting an order, $\frac{\lambda}{2}(1-\bar{\beta}) - c$, does not vary over time. As a result, to keep the speculator indifferent between trading and not as time passes, continuation profits from deferring must not fall even though fewer opportunities remain in which to benefit from any price improvement gained by deferring. Hence, as time passes, prices must fall by more (i.e., market maker beliefs must be revised downward by more) following an order of zero. In turn, more dramatic price revisions require that the speculator trade more aggressively on information as date T approaches, so that an order flow of zero conveys more information.

We now extend lemma 1 to the general period i case. When the probability of information leakage jumps in a non-convex way, it is difficult to rule out the possibility of multiple solutions to (4) and hence the possibility that the speculator's trading intensity *rises* with an increase in the market maker's prior. To preclude this possibility, we assume that

Assumption (A1): $\chi_i(\beta) > 0$ implies that $\chi_j(\beta) > 0, \forall j < i$.

Proposition 1 ensures that **A1** is satisfied if $\lambda_i = \lambda, \forall i$. Assumption **A1** precludes multiple solutions to (4). It allows us to extend lemma 1 and characterize the speculator's equilibrium trading strategy at all dates. The next proposition details that the speculator is less likely to trade when the market maker believes the good state is more likely to occur:

Proposition 2 *The equilibrium probability with which the speculator submits an order, $\chi_i(\beta_{i+1})$, declines monotonically in β_{i+1}; strictly decreasing with β_{i+1} for $\chi_i(\beta_{i+1}) \in (0, 1)$.*

This result is subtle. Period trading profits are smaller when the market maker believes the good state is more likely. However, future market maker beliefs about the good state will also be higher if information is not revealed, so that future trading profits are also lower. Hence, to prove proposition 2, we must show that current profits fall more rapidly with β than do continuation profits. **A1** ensures that continuation payoffs following an order flow of zero are a concave function of β, while trading profits are a linear function of β. Then, *ceteris paribus*, increasing β reduces the value the speculator places on trading relative to manipulating market maker beliefs. To keep the speculator indifferent, market maker beliefs must fall by less after an order flow of zero. Hence, the speculator must be less likely to trade.

Corollary 1 describes when the speculator submits an order with positive probability.

Corollary 1 *A sufficient condition for there to be a non-trivial range of market maker beliefs, for which the speculator trades with positive probability at period i is:*

$$\lambda_i - 2c > \sum_{j=1}^{i-1} \left(2^{j-i}(\lambda_j - 2c) \left[\prod_{k=j+1}^{i} (1-\lambda_k) \right] \right)$$

and $\lambda_1 > 2c$. If $\lambda_i = \lambda$, $\forall i$, then the sufficient condition reduces to $\lambda > 2c$ — the likelihood that the private information is revealed in the next period must exceed twice the fixed trading costs as a share of the maximum contract value.

Propositions 3–5 characterize how the parameters describing the economy affect trading:

Proposition 3 *The more likely the speculator's information is to be revealed publicly through a leak, the more aggressively the speculator trades: χ_i is weakly increasing in λ_i, strictly increasing for $\chi_i \in (0,1)$. If $\lambda_i = \lambda \; \forall i$, then χ_i is weakly increasing in λ.*

As λ_i rises, the speculator's private information is more likely to be impounded into the closing period i price, raising the expected return from a period i contract. Further, an increase in λ_i reduces the expected gain to deferring from trade, as the speculator's information is less likely to remain private. Less obviously, this relation still holds when the probability that the private information is revealed does not vary with time. There are three direct effects: an increase in λ, (1) increases the speculator's expected return from submitting an order this period; and (2) reduces the expected return from deferring since it increases the probability that information will be revealed before next period; but the potentially offsetting effect, is (3) if the information is not revealed this period, the expected return from submitting an order next period rises. We show, however, that the first two effects dominate the third.

Proposition 4 *The greater are fixed trading costs, the less aggressively the speculator trades: χ_i is decreasing in c, strictly decreasing for $\chi_i \in (0,1)$.*

Higher fixed trading costs (c) reduce the expected return from submitting an order. When c is high, the benefit from trading is low unless the price deviates substantially from the true value. As c rises, the speculator is more likely to defer from trading, hoping to obtain lower prices next period, which would raise the net profit margin.

Proposition 5 *The more likely a speculator is to have private information about the risky asset, the less aggressively he trades:* χ_i *is weakly decreasing in* δ*, strictly decreasing for* $\chi_i \in (0, 1)$.

As δ increases, the market maker believes that there is more likely to be a speculator, so her beliefs about the likelihood of a good state fall more sharply following an order flow of zero. Hence, as δ increases, it becomes more profitable for the speculator to defer from trading.

4 Time-to-Maturity

Until now, we have considered only short-dated contracts that expire after one period. In practice, options and futures contracts exist for various lengths of time. As a contract's time-to-maturity increases, the model tends to that of a long-lived trader in equity (Kyle 1985), with agents able to hold positions for as long as it takes the market maker to learn the asset's true value. In contrast to Kyle, in our model speculators may initially refrain from trading when they receive (private) news: if the price of the asset is too close to its fundamental value, a speculator prefers to defer in the hope of manipulating the market. This result reflects that our model allows for non-convexities — fixed trading costs and round lot restrictions. Kyle's normality assumptions preclude consideration of these important non-convexities, making submitting a sufficiently small order more attractive than completely delaying trade.

Our novel framework allows us now to investigate two previously unexplored aspects of how a contract's time-to-maturity affects outcomes. Section 4.1 details that if contracts exist for multiple periods then the speculator's accumulated position affects his trading strategy. Section 4.2 provides an explanation for why markets for shorter-term contracts are far more liquid than those for longer-term contracts.

4.1 Accumulated Position

A speculator with an accumulated position takes into consideration that submitting an order for a contract raises the probability that his information will be revealed, in which case his existing contracts will then be settled for a profit. This situation does not arise with equity because, as long as a speculator can hold his stake, his information will eventually be incorporated into the price of a previously-accumulated position. In contrast, a short-term contract may expire before the information is revealed publicly. Consequently, increasing the probability that the information is revealed has a positive value, a value that rises with the speculator's accumulated position.

Proposition 3 implies that the longer is the holding period, the more aggressively the speculator trades. That is, raising λ captures the effects of a longer holding period, ignoring the impact of a speculator's accumulated position on his trading behavior. We now document in the simplest possible context, that a speculator's accumulated position further increases his trading intensity.

Consider the last two periods remaining for a long-dated contract written at the beginning of period T and expiring at the end of period 1. We assume that $\lambda_1 < 1$, so that the speculator's accumulated position could expire before his information is fully incorporated into prices. Each period, market participants can purchase contracts that expire at the end of period 1. Each period, with probability $\frac{1}{2}$, a liquidity trader places an order.

The speculator's accumulated position at the end of period $i+1$ is $\sum_{\iota=i+1}^{T} X_\iota$. His period i trading strategy is a probability distribution Pr $\left\{X_i = x_i \mid v, \mathbf{H}_{i+1}, \mathbf{\Theta}_{i+1}, \sum_{\iota=i+1}^{T} X_\iota\right\}$ over feasible order sizes which depends on this accumulated position. In equilibrium, any informed order of size $X_i \notin \{0,1\}$ reveals the speculator's information to the market maker, in which case the price reflects his information. Hence, we need only consider three period trading strategies for the speculator: (1) submit an order of size two that reveals his information; (2) submit an order of size one; (3) defer from trading. The period i value function is:

$$V_i\left(\mathbf{H}_{i+1}, \sum\nolimits_{\iota=i+1}^{T} X_\iota\right) = \max_{X_i \in \{0,1,2\}} E\left[\pi_i\left(Y_i|\mathbf{H}_{i+1}, \sum\nolimits_{\iota=i+1}^{T} X_\iota\right) + (1-\lambda_i)V_{i-1}\left(\mathbf{H}_i(X_i), \sum\nolimits_{\iota=i}^{T} X_\iota\right)\right],$$

where $V_0 = 0$ and the associated expected period i payoffs are:

$$E\left[\pi_i\left(X_i\middle|\mathbf{H}_{i+1}, \sum_{\iota=i+1}^{T} X_\iota\right)\right] = \begin{cases} \left(\sum_{\iota=i+1}^{T} X_\iota\right) - c & \text{if } X_i = 2 \\ \frac{1}{2}\left(\lambda_i - P_i\left(Y_i = 1|\mathbf{H}_{i+1}\right)\right) - c + \left(\frac{1+\lambda_i}{2}\right)\sum_{\iota=i+1}^{T} X_\iota & \text{if } X_i = 1 \\ \lambda_i \sum_{\iota=i+1}^{T} X_\iota & \text{if } X_i = 0 \end{cases}$$

Proposition 6 *For any given market maker beliefs, β_i, the expected size of the speculator's order in period i, $i = 1, 2$ increases with his accumulated position.*

Proposition 6 reveals that holding both a speculator's information and total past (informed plus liquidity) trade constant, future expected speculative trade and hence future expected volume is greater when the speculator has a greater share of past trade. That is, the market maker updates prices in the same manner independently of who submitted the orders, but the speculator trades more aggressively in the future has a greater stake due to more aggressive trading in the past.

4.2 Liquidity of Distant Contract Markets

This section investigates why option and futures contracts that are relatively close to maturity are far more liquid than similar contracts with more distant expiration dates. This empirical regularity is especially puzzling given the many reasons why agents might want to use longer-term contracts to hedge against long-term risk. To do this, we extend the two-period model as follows:

- *Two* contracts are available each period: a 1-period (*short-dated*) contract and a 2-period (*long-dated*) contract. The market maker sees total order flows in *both* markets.
- As in the basic model, each period an *exogenous* short-lived liquidity trader arrives with independent probability $\frac{1}{2}$ and trades only once. Now, unlike the basic model, this trader randomly submits an order for either the long-dated contract or the short-dated contract with equal probability. This structure ensures that the exogenous probability of liquidity trade is the same in each contract market.
- At the beginning of period 2, a *long-lived* liquidity trader, called the "hedger", arrives with independent probability $\frac{1}{2}$. The trading behavior of this trader is *endogenous*: he can hedge against an income shock that is negatively correlated with the good state in one of two ways: (1) buy a long-dated contract at period 2; (2) buy a short-dated contract at period 2, and if uncertainty about the state is not resolved, purchase a short-dated contract at period 1. Let ξ be the probability that the hedger buys a long-dated contract. The trader selects the hedging strategy that minimizes his expected cost.

Thus, each period there are three potential traders: an informed speculator, an exogenous liquidity trader, and an endogenous liquidity trader (the hedger). To focus on the long-term composition of trade in long- and short-dated contracts, we assume that private information is revealed with certainty at the end of period 1, $\lambda_1 = 1$. As a result, the speculator has no incentive to submit a large order in period 1 in order to reveal information. This allows us to focus on the choice between trading contracts of different maturities without complicating the analysis with the issues related to the speculator's accumulated position that we detail in section 4.1.

We index short-dated contract variables by S and long-dated contract variables by L. Bold symbols denote vectors: for example, $\mathbf{Y}_i = (Y_i^S, Y_i^L)$ denotes total order flows in period i for short- and long-dated contracts. The history of total order flow through period i is: $\mathbf{H}_i = \{Y_T^S, Y_{T-1}^S, ..., Y_i^S;\ Y_T^L, Y_{T-1}^L, ..., Y_i^L\}$. The speculator's period i strategy is a joint probability distribution over feasible orders, Pr$\{\mathbf{X}_i = (x_i^S, x_i^L) \mid v, \mathbf{X}_{i+1}, \mathbf{H}_{i+1}, \mathbf{\Theta}_{i+1}\}$. The market maker selects a set of pricing functions for each contract at the open and close, $P_{io}^j(\mathbf{H}_i, \mathbf{\Theta}_{i+1})$, $P_{ic}^j(\mathbf{H}_i, \mathbf{\Theta}_i)$, $j = S, L$. The price for each contract depends on order flow in *both* markets. Because order flow at period 2 provides the market maker information about the presence of a hedger, the market maker's updating rule at period 1 depends distinctly on both her original prior and observed past order flow.

Since $\lambda_1 = 1$, orders received at period 1 for the long- and short-dated contracts are equivalent ($X_1 = X_1^S = X_1^L$). The speculator's value function in the last period is:

$$V_1(\mathbf{Y}_2, \mathbf{X}_2) = \max_{X_1} E\left[\pi_1(X_1, \mathbf{Y}_2, \mathbf{X}_2)\right]$$
$$= \max_{X_1} \begin{cases} X_2^L & \text{if } X_1 = 0 \\ X_2^L + \frac{(1-\Sigma)}{2}(2 - \beta_1(1, (\mathbf{Y}_2)) - \beta_1(2, (\mathbf{Y}_2))) + \frac{\Sigma}{2}(1 - \beta_1(2, (\mathbf{Y}_2))) - c & \text{if } X_1 = 1 \\ X_2^L + (1-\Sigma)(1 - \beta_1(2, (\mathbf{Y}_2))) - c & \text{if } X_1 = 2 \\ X_2^L - c & \text{if } X_1 > 2, \end{cases}$$

where $\Sigma = \Sigma(\mathbf{Y}_2, \mathbf{X}_2)$ is the probability with which the speculator believes a hedger purchased the short-dated contract.[10]

The possibility of two liquidity traders in either market means that the speculator can submit an order of size two, or submit orders to both markets, without being revealed for sure to the market maker. However, lemmas 4 and 5 in the appendix show that it is never optimal for the speculator to

[10] If $Y_2^S + Y_2^L \geq 3$, private information is revealed with certainty. For $Y_2^S + Y_2^L < 3$, $\Sigma((0,\cdot),(\cdot,\cdot)) = \Sigma((1,0),(1,0)) = \Sigma((1,1),(1,1)) = \Sigma((1,1),(1,0)) = \Sigma((2,0),(2,0)) = 0$; $\Sigma((1,0),(0,0)) = \Sigma((1,1),(0,1)) = \Sigma((2,0),(1,0)) = (1-\xi)/((1-\xi)+\frac{1}{2})$; $\Sigma((1,1),(0,0)) = 1-\xi$; $\Sigma((2,0),(0,0)) = 1$.

do so. This result holds in period 1 *even* though the speculator may know from the period 2 order flow net of his trade that there is no hedger, so that the maximum liquidity trade is one, whereas the market maker cannot make such a distinction. Restricting attention to the speculator's three possible equilibrium period 2 trading strategies, his period 2 value function is:

$$V_2 = \max_{\mathbf{X}_2} E\left[\pi_2\left(\mathbf{X}_2\right) + (1-\lambda_2)V_1\left(\mathbf{Y}_2, \mathbf{X}_2\right)\right],$$

where $E\left[\pi_2\left(\mathbf{X}_2\right)\right] =$

$$\begin{cases} 0 & \text{if } \mathbf{X}_2 = (0,0) \text{ (defer from trading)} \\ \lambda_2 - \frac{1}{8}[2 + (1+2\xi)P_2^L(0,2) + (3-2\xi)P_2^L(1,1) + 2P_2^L(0,1)] - c & \text{if } \mathbf{X}_2 = (0,1) \text{ (buy one long-dated)} \\ \lambda_2 - \frac{1}{8}[2 + (1+2\xi)P_2^S(1,1) + (3-2\xi)P_2^S(2,0) + 2P_2^S(1,0)] - c & \text{if } \mathbf{X}_2 = (1,0) \text{ (buy one short-dated)} \end{cases}$$

Figure 2 illustrates the period 2 strategies from the market maker's perspective. As before, the market maker updates her beliefs in response to different order flows using Bayes' Rule. Derivations of these updating rules and the resulting hedger's expected trading costs can be found in our working paper available at *http://ssrn.com/abstract=443360*.

Analytical characterizations are difficult because of the interaction between market maker beliefs and the strategies of the endogenous hedger and the speculator. This leads us to describe outcomes numerically. Figure 3 presents a surface diagram of the percentage of trade in long-dated contracts, for $c = 0.05$ and different values of δ and λ_2. For most values of δ or λ_2, most trade occurs in short-dated contracts. Long-dated contracts draw a majority of trade only when both δ is very small (private information is unlikely) and λ_2 is very small (so a hedger is still likely to need to hedge at period 1). Short-dated contracts draw most trade *despite* relatively high fixed costs, which increase the relative cost of rolling over nearby contracts. Similar outcomes hold for values of $c \leq 0.05$. Note that $c = 0.05$ represents fixed trading costs equal to 5% of the maximum value of the contract, and more than 10% of the maximum potential profit—higher fixed costs are unreasonable. Thus, we find that for reasonable parameterizations, trade is concentrated in nearby contracts.

To understand better why trade concentrates in short-dated contracts in equilibrium, we explore how a hedger's expected total trading cost varies as we *exogenously* alter the probability with which he submits an order for the long-dated contract, and then compute the consistent optimal informed trading strategies and "equilibrium" zero-profit pricing. Panel A of Figure 4 shows that for almost all realistic parameter ranges, the hedger's expected costs are minimized if he primarily trades short-dated contracts. Panel B provides the economic intuition: as liquidity trade, ξ, in long-dated contracts is raised, the speculator trades the long-dated contract with increasing probability (given the "equilibrium" pricing associated with higher liquidity trade).

Long-dated contracts allow the speculator either to accumulate a larger position or to trade less frequently (reducing the probability of being uncovered), so that adverse selection costs faced by a hedger tend to rise with the probability that he hedges using the long-dated contract. Notice, however, that the hedger's expected costs are not minimized by always buying the short-dated contract ($\xi = 0$). This is because a small increase in ξ, say from 0 to 0.03, "raises" the (consistent) likelihood with which the informed speculator trades the long-dated contract from 0.303 to 0.369. This highlights the tradeoff faced by the hedger: raising ξ improves prices in the short-dated contract by reducing adverse selection costs in those contracts; but raising ξ also "causes" the hedger to buy more long-dated contracts, which are more expensive (speculative trade in these contracts is more likely). The added cost of the long-dated contracts quickly dominates as ξ rises.

Figure 4 reflected parameterizations for which the speculator always traded in period 2, mixing only over which contract to trade. Now consider parameterizations where the speculator may defer from trading at period 2 in equilibrium, such as when λ_2 is small. In this situation, the speculator mixes between deferring from trade and submitting an order for the long-dated contract. Now, if the hedger increases the frequency with which he trades the short-dated contract (i.e., reduces ξ), this may cause the speculator to raise the probability that he defers from trading. Thus, for these parameterizations, the hedger often optimally trades *only* the short-dated contract.

Panel A of Figure 5 shows how the hedger's mixing probability varies with the fixed trading cost, c. This clearly illustrates the hedger's preference for a "pooling" equilibrium outcome. For small c, the hedger almost always buys the short-dated contract. As c increases, he first begins to trade slightly more of the long-dated contract, but as c rises marginally further, he dramatically changes his strategy and trades long-dated contracts almost exclusively. Intuitively, once c is sufficiently high, the added trading cost from buying two short-dated contracts exceeds the higher adverse selection costs in the long-dated contract.

Panel B of Figure 5 shows that, corresponding with the dramatic change in the hedger's behavior revealed in Panel A, there is a large jump in the percentage of trade in long-dated contracts when trading costs reach a critical level. For c less than this critical level, the percentage of trade in long-dated contracts rises with the fixed trading cost c, as one might expect. This pattern occurs because private information is likely to be revealed at period 2 ($\lambda_2 = 0.8$).

In contrast, Figure 6 details outcomes when $\lambda_2 = 0.2$ so that private information is unlikely to be revealed at period 2. This figure reveals that trade in long-dated contracts need not always rise with c. Again, for a range of c, we observe the jump corresponding to the change from pooling in short-dated contracts to long-dated contracts. But, unlike before, if c is small, raising c *reduces* trade in long-dated contracts. It is possibilities such as this that preclude analytical characterizations. Indeed, neither total expected trade across contracts, nor total expected trade in the long-dated contract need be monotone in c. To understand why, consider first how liquidity trade varies with c, and then consider how speculative trade varies.

The hedger's strategy, ξ, is generally insensitive to small changes in fixed trading costs, because he prefers to trade almost exclusively in one of the two contracts. From the hedger's perspective, increasing c raises the attractiveness of trading long-dated contracts in terms of transaction costs. When c is low, this benefit fails to offset the far higher adverse selection costs associated with moving away from a pooling equilibrium in short-dated contracts toward trading both short- and long-dated contracts, nor is it sufficient to prompt the hedger to trade only the long-dated contracts. There is a very narrow range of c over which transaction costs dominate adverse selection costs and the hedger switches from almost always trading short-dated contracts to almost always trading long-dated contracts. Save for this range of fixed costs, changes in the share of trade in short- and long-dated contracts are driven by changes in the speculator's behavior.

The speculator's behavior depends critically on the likelihood his information is to be revealed at period 2. Consider fixed trading costs c for which the hedger almost always trades the short-dated contract and consider the impact of raising c for:

(a) Information likely to be revealed: When λ_2 is high, the speculator always trades at period 2, mixing between the short- and long-dated contracts. The high probability of information leakage at period 2 raises the attractiveness of trading, reducing the distinction between short- and long-dated contracts. When fixed transaction costs rise, the speculator switches from trading short-dated contracts to trading long-dated contracts. As a result, the share of trade in long-dated contracts *rises*.

(b) Information unlikely to be revealed: When λ_2 is sufficiently low, then independent of market maker beliefs, the speculator never optimally submits a period 2 order for the short-dated contract (the contract is likely to expire worthlessly). When fixed transaction costs rise, the speculator switches from trading long-dated contracts to deferring from trade at period 2. The net effect is that the share of trade in long-dated contracts *falls*.

Thus, higher fixed trading costs may not increase the share of trade in long-dated contracts. Finally, to understand why total expected trade need not fall as fixed trading costs rise, suppose that λ_2 is small, and c is low enough that liquidity traders trade the short-dated contract. Then as c rises, the speculator increasingly defers from trading (as information is unlikely to be revealed), reducing total expected trade. But eventually, c rises by enough that hedgers switch to trading long-dated contracts. Once c is high enough that hedgers almost always trade long-dated contracts, adverse selection costs for these contracts fall so that it suddenly becomes profitable for the informed speculator to trade long-dated contracts heavily; and as a result total trade rises.

Our characterizations reveal that for reasonable levels of fixed trading costs, long-dated contract markets are likely to thrive only in environments where informational asymmetries are slight (e.g. weather derivatives, but not equity derivatives). Of course, institutional factors other than information asymmetry may also influence liquidity in specific markets. Thus, predictions are cleanest in settings where we can control for these other variables—for instance, comparing liquidity in long-dated contracts for stocks with significant information asymmetry versus those with less.

The recent introduction of long-dated contracts in a variety of markets is in an attempt to satisfy market participant demand for hedging long-term risk. Despite active promotion, most long-dated contract markets are still highly illiquid. If trading venues hope to influence equilibrium outcomes by changing fixed trading costs, then the impact on liquidity may be more subtle than they might anticipate. To whit, our results show that when information is likely to remain privately held, increasing relative trading costs in short-dated contracts may *raise* trade in short-dated contracts because it reduces the attractiveness to speculators of trading on long-term information, thereby lowering adverse selection costs.

Trade in underlying equity: We conclude by elaborating on the issues involved in allowing for trade in the underlying equity. In our "simple" 2-period model, all outstanding information is revealed in period 1 with certainty – thus, in period 2, traders must choose between a short-dated contract which might expire worthlessly and a long-dated contract which does not have this risk. In other words, trading the long-dated contract is, for all effective purposes, equivalent to trading the underlying equity in the context of our model.[11] By allowing individuals to choose between a short-dated contract and a long-dated "equity-like" contract, our model comes closer than *any existing model* to representing, in a dynamic context, concurrent trade in equity and derivative markets — an essentially unsolved problem in the theoretical market microstructure literature.[12]

That said, our model is best interpreted as only one of short-lived securities. The reason is that we consider only one piece of information against which liquidity traders face adverse selection. In particular, an unmodeled cost in our environment is that there will generally be new information arrival over time against which individuals will incur adverse selection costs each time they trade. Assuming one informational event is reasonable if an agent's trading horizon is not too long, but increasingly misses reality as the horizon gets longer.

To see why, consider an environment with *multiple* information arrival occurrences. Here, at any given time, most speculators only have advance knowledge of a subset of the information that will eventually arrive; they will acquire and exploit the remaining information in the future. Market makers will incorporate into their prices the adverse selection costs of the *nearby* information asymmetry—the rest does not exist yet and will not exist during the market maker's relatively short holding horizon. In this environment, unlike that modeled in our paper, a liquidity trader with a long trading horizon has an additional reason to trade equity. Specifically, at the time of trading, a liquidity trader in equity will only bear the adverse selection costs for some of the potential

[11] Of course, if trading the long-dated contract is not equivalent to trading the underlying equity then it is possible that the relaxation of the assumption restricting trading in the underlying equity could alter the results of this model.

[12] Note, our model differs from that of existing models of hedging – here we model explicitly how a trade in one market impounds information into prices in both markets, allowing us to model dynamic trading behavior. In contrast, prices are exogenous in standard hedging models.

information events over which he is potentially asymmetrically informed. In contrast, a liquidity trader who repeatedly rolls over short-dated contracts must pay for the adverse selection costs of virtually all of the new informational asymmetries as they arise over time. Because our model only allows for asymmetry on a single piece of information, our model is most suitable for explaining preferences between two relatively short-term trading strategies, such as explaining why farmers often elect to hedge their crops by rolling over two 2-month futures contracts instead of buying a single 4-month futures contract that matches their horizon.

While our model works best in the context of a short-term trading horizon, we believe that it can provide important insights into unraveling the complex pattern of trade in equity and in derivatives of many different time-to-maturities. We argue that additional information arrival raises the cost of trading short-dated contracts relative to that of long-dated contracts and equity. The extent to which this is so depends on the trading horizons of the liquidity traders. If the horizons over which an agent wants to hold a position are relatively short, rolling over short-dated contracts will continue to be the optimal course of action; while with longer holding horizons, agents are best off holding equity. Thus, liquidity in each of these markets will depend on empirical questions such as (i) what is the degree of segmentation between the derivative market and the underlying market?; (ii) what is the rate of information arrival?; (iii) how heterogeneous are the information sets of market participants?; and (iv) what is the composition of trade between liquidity- and information-based motives? Current research is only just beginning to uncover the answers to these important empirical questions (see, e.g., Chakravarty, Gulen, & Mayhew, 2004).

5 Conclusion

Despite its importance, strategic trading of short-lived securities, such as option or futures contracts, has largely been ignored by the academic literature. This paper documents important differences between the strategic trading of short-lived securities and that of equity:

1. The shorter horizon in which information must be impounded for a short-lived security to pay off makes an informed speculator more reluctant to trade, especially when the speculator's informa-

tion is longer-term in nature. Given innocuous technical conditions, speculative trading intensities rise over the trading horizon so that prices become more sensitive to order flow.

2. With short-lived securities, the greater a risk neutral speculator's holdings of the short-lived security, *ceteris paribus*, the more aggressively he trades in the future. In contrast, in equity markets, a speculator's accumulated position does not affect his trading behavior.

3. For reasonable parameter ranges, liquidity traders prefer to incur extra costs to roll over their short-term positions rather than trade in distant contracts, precisely because equilibrium adverse selection costs are smaller in shorter-contracts. This allows us to reconcile the puzzling empirical finding that markets for longer-term contracts have little liquidity and large spreads.

6 Appendix: Proofs

Proof of Lemma 1: Let LHS and RHS be the values of the left- and right- hand sides of equation (4). Clearly, $LHS < 0$ for $\beta_3 > 1 - \frac{2c}{\lambda_2}$. Since $RHS \geq 0$, any solution to (4), if one exists, must occur for $\beta_3 < 1 - \frac{2c}{\lambda_2}$. Intuitively, the speculator only submits an order with positive probability if the expected one-period return from doing so is positive. We know: (a) $\beta_2(\beta_3, Y_2 = 0) < \beta_3$; (b) $\lambda_1 \geq \lambda_2$. Hence, for $\beta_3 \in \left(0, 1 - \frac{2c}{\lambda_2}\right)$, it must be that $\frac{\lambda_1}{2}(1 - \beta_2(\beta_3, Y_2 = 0)) - c > 0$ and the RHS can be expanded as:

$$(1 - \lambda_2)\left[\frac{\lambda_1}{2}(1 - \beta_2(\beta_3, Y_2 = 0)) - c\right]. \quad (5)$$

Holding χ_2 constant, the first derivative of (5) w.r.t. β_3 is $-\frac{\lambda_1(1-\lambda_2)(1-\chi_2\delta)}{2(1-\chi_2\delta\beta_3)^2} < 0$ and the second derivative of (5) w.r.t. β_3 is $-\frac{\lambda_1(1-\lambda_2)\chi_2\delta(1-\chi_2\delta)}{(1-\chi_2\delta\beta_3)^3} < 0$. Thus, RHS is strictly concave for $\beta_3 \in \left(0, 1 - \frac{2c}{\lambda_2}\right)$. The derivative of LHS w.r.t. β_3 is $-\lambda_2 < 0$. Hence, there is at most one solution to (4) for $\beta_3 \in [0, 1]$. Holding χ_2 "fixed" at a solution to (4) at period 2, it follows that LHS must fall more quickly with an increase in β_3 than RHS. Since LHS is independent of χ_2 and RHS rises with χ_2, to preserve equality, the mixing probability χ_2 must fall with β_3, falling strictly for $\chi_2 \in (0, 1)$. ■

Lemma 2 *If* $\lambda_i = \lambda \ \forall i$, *then* $V_i(\beta) \geq V_{i-1}(\beta) \ \forall \beta \in [0, 1], \ \forall i$.

Proof of Lemma 2: Case 1: If $\beta \geq 1 - \frac{2c}{\lambda}$, then $V_i(\beta) = (1-\lambda)^{i-1}V_1(\beta) = 0$. Then $V_i(\beta) = V_{i-1}(\beta)$. Case 2: If $\beta < 1 - \frac{2c}{\lambda}$, then $V_i(\beta) = \frac{(1-\lambda)}{2}V_{i-1}(\beta) + \frac{1}{2}[\lambda(1-\bar{\beta}) - 2c]$ and $V_{i-1}(\beta) = \frac{(1-\lambda)}{2}V_{i-2}(\beta) + \frac{1}{2}[\lambda(1-\bar{\beta}) - 2c]$. It follows that $V_i(\beta) > V_{i-1}(\beta)$ iff $V_{i-1}(\beta) > V_{i-2}(\beta)$. Since $V_1 = \frac{1}{2}[\lambda(1-\bar{\beta}) - 2c] > V_0 = 0$, the result follows from induction.■

Proof of Proposition 1: The mixing probability $\chi_i(\bar{\beta})$ solves $\lambda(1-\bar{\beta}) - 2c = (1-\lambda)V_{i-1}(\beta_i(\bar{\beta}, 0))$. The proposition has two parts (A and B):

Part A: We show that $\bar{\beta} < 1 - 2c\lambda^{-1}$ is *necessary* and *sufficient* for $\chi_i(\bar{\beta}) > 0$.

Necessity: $\bar{\beta} \geq 1 - 2c\lambda^{-1} \Rightarrow \lambda(1-\bar{\beta}) - 2c < 0$. Since $V_{i-1}(\cdot) \geq 0\ \forall i$, the speculator defers.

Sufficiency: Proof by induction. In **period 1**, if $\bar{\beta} < 1 - \frac{2c}{\lambda}$, then $\frac{\lambda}{2}(1-\bar{\beta}) - c > 0$ and the speculator submits an order. In **period 2,** if $\bar{\beta} < 1 - \frac{2c}{\lambda}$ and $\chi_1(\bar{\beta}) = 1$, then $\lambda(1-\bar{\beta}) - 2c > (1-\lambda)V_1(\bar{\beta}) = \frac{(1-\lambda)}{2}[\lambda(1-\bar{\beta}) - 2c]$. That is, if the market maker *believes* that the speculator will defer, then the speculator's expected return from submitting an order exceeds his expected continuation payoff. Thus, in equilibrium, $\chi_2(\bar{\beta}) > 0$. In an **arbitrary period i,** if $\bar{\beta} < 1 - \frac{2c}{\lambda}$ and $\chi_j(\bar{\beta}) > 0\ \forall j < i$, then $\lambda(1-\bar{\beta}) - 2c > (1-\lambda)V_{i-1}(\bar{\beta}) = [\lambda(1-\bar{\beta}) - 2c]\sum_{p=1}^{i-1}\left[\frac{(1-\lambda)}{2}\right]^p$, since $1 > \sum_{p=1}^{i-1}\left[\frac{(1-\lambda)}{2}\right]^p$. Hence, $\chi_i(\bar{\beta}) > 0$.

Part B: Three cases. **Case 1:** If $\chi_i(\bar{\beta}) = 1$, then it follows immediately that $\chi_i(\bar{\beta}) \geq \chi_{i+1}(\bar{\beta})$. **Case 2:** If $\chi_i(\bar{\beta}) = 0$, then from part A it follows that $\lambda(1-\bar{\beta}) - 2c < 0$ and $\chi_{i+1}(\bar{\beta}) = 0$. **Case 3:** If $\chi_i(\bar{\beta}) \in (0,1)$ then either $\chi_{i+1}(\bar{\beta}) \in (0,1]$ or $\chi_{i+1}(\bar{\beta}) = 0$, then $(1-\lambda)V_{i+1}(\beta_i(\bar{\beta},0)) = (1-\lambda)V_i(\beta_{i+1}(\bar{\beta},0)) = \lambda(1-\bar{\beta}) - 2c$. Hence, $V_{i+1}(\beta_i(\bar{\beta},0)) = V_i(\beta_{i+1}(\bar{\beta},0))$. From lemma 2, this occurs only if $\beta_i(\bar{\beta},0) < \beta_{i+1}(\bar{\beta},0)$. Since $\frac{\partial\beta_i(\bar{\beta},0)}{\partial\chi_i} \leq 0$, it follows that $\chi_i(\bar{\beta}) > \chi_{i+1}(\bar{\beta})$. ■

Proof of Proposition 2: Interior values for period i trading probabilities $\chi_i(\beta_{i+1})$ must solve equation (4). Let $LHS(i)$ and $RHS(i)$ be the period i values of the left- and right- hand sides of (4). Clearly, $LHS(i) < 0$ for $\beta_{i+1} > 1 - \frac{2c}{\lambda_i}$. Since $RHS(i) \geq 0$, any solution to (4) must occur for $\beta_{i+1} < 1 - \frac{2c}{\lambda_i}$. We know: (a) market maker beliefs are non-increasing in response to aggregate order flows of 0 or 1 ($\beta_i(\beta_{i+1}, Y_i \in \{0,1\}) < \beta_{i+1}\ \forall i$); and (b) the probability of information leakage is non-decreasing, ($\lambda_i \geq \lambda_{i+1},\ \forall i$). Hence, for $\beta_{i+1} \in \left(0, 1 - \frac{2c}{\lambda_i}\right)$,

$$\frac{\lambda_i}{2}\left(1 - \beta_j\left(\beta_{i+1}, \{Y_j, Y_{j+1}, ..., Y_i\}\right)\right) - c > 0,\ \forall j < i. \tag{6}$$

Using an induction argument and **A1**, we now show that if χ_j declines with β_{j+1}, $\forall j < i$, then χ_i falls with β_{i+1}. The result for periods 1 and 2 is immediate. χ_1 must fall with β_2 since $\chi_1(\beta_2) = 1$ for $\beta_2 < 1 - \frac{2c}{\lambda_1}$; $\chi_1(\beta_2) \in [0, 1]$ for $\beta_2 = 1 - \frac{2c}{\lambda_1}$; and $\chi_1(\beta_2) = 0$ for $\beta_2 > 1 - \frac{2c}{\lambda_1}$. We also showed in lemma 1 that χ_2 fell with β_3.

Period 3: For $\beta_4 \in \left(0, 1 - \frac{2c}{\lambda_3}\right)$, using (6) we expand $RHS(i = 3)$ as:

$$\begin{aligned} &\frac{(1-\lambda_3)}{2}\Big[(1-\lambda_2)\left(\frac{\lambda_1}{2}(1-\beta_3(\beta_4,0)) - c\right) \\ &+ \max\left\{\lambda_2(1-\beta_3(\beta_4,0)) - 2c,\ (1-\lambda_2)\left(\frac{\lambda_1}{2}\left(1-\beta_2(\beta_3(\beta_4,0),0)\right) - c\right)\right\}\Big]. \end{aligned} \tag{7}$$

Let $\beta_4 \in (0, \beta_4^*)$ denote the range of β_4 such that $\chi_3(\beta_4) > 0$. From **A1**, if $\chi_3(\beta_4) > 0$, then $\chi_2(\beta_4) > 0$. Further $\chi_3(\beta_4) > 0$ implies $\chi_2(\beta_3(\beta_4, 0)) > 0$ because $\beta_3(\beta_4, 0) \leq \beta_4$ and $\chi_2(\beta)$ is falling in β. Thus, we need only consider (7) for β_4 corresponding to $\chi_2(\beta_3(\beta_4, 0)) > 0$.

Since $LHS(i = 3) > RHS(i = 3)$ for $\beta_4 \geq 1 - \frac{2c}{\lambda_3}$, **A1** implies that if $\chi_3(\beta_4) > 0$ then $LHS(3) > RHS(3)$ for $\beta_4 > \beta_4^*$. Given these observations and that the derivative of $LHS(3)$ w.r.t. β_4 is constant and equal to $-\lambda_3 < 0$, a sufficient condition to ensure at most one solution exists to (4) evaluated at period 3 is that

$$(1-\lambda_3)\left[\frac{(1-\lambda_2)}{2}\left(\frac{\lambda_1}{2}(1-\beta_3(\beta_4,0)) - c\right) + \frac{\lambda_2}{2}(1-\beta_3(\beta_4,0)) - c\right] \tag{8}$$

be strictly concave. (8) corresponds to $RHS(3)$ evaluated over $\beta_4 \in (0, \beta_4^*)$. Holding χ_3 constant, the first derivative of (8) w.r.t. β_4 is $-(1-\lambda_3)\left[\frac{(1-\lambda_2)\lambda_1}{4} + \frac{\lambda_2}{2}\right]\frac{\partial\beta_3(\beta_4,0)}{\partial\beta_4} < 0$ and the second derivative is $-(1-\lambda_3)\left[\frac{(1-\lambda_2)\lambda_1}{4} + \frac{\lambda_2}{2}\right]\frac{\partial^2\beta_3(\beta_4,0)}{\partial\beta_4^2} < 0$. Thus $RHS(3)$ is strictly concave for $\beta_4 \in (0, \beta_4^*)$ and there is at most one solution to (4) at period 3. Holding χ_3 "fixed" at a solution to (4) at period 3, it follows that $LHS(3)$ falls more quickly with an increase in β_4 than $RHS(3)$. To preserve equality, since $LHS(3)$ is independent of χ_3 and $RHS(3)$ rises with χ_3, the mixing probability χ_3 must fall with β_4, strictly so if $\chi_3 \in (0, 1)$.

Period i: By induction. Suppose the informed's mixing probability for periods $j < i$, falls with β_{j+1}. Using the same logic as for period 3, we show that if $\chi_i(\beta_{i+1}) > 0$, then $\chi_j(\beta) > 0,\ \forall\beta \leq$

β_{i+1}, $\forall j < i$. Let β^*_{i+1} be the maximum β_{i+1} such that $\chi_i(\beta_{i+1}) > 0$. Hence, possible solutions for the period i analog of equation (4) are in some range $\beta_{i+1} \in (0, \beta^*_{i+1})$. Expanding $RHS(i)$ yields,

$$\sum_{p=1}^{i-1} \left(2^{p-i}(\lambda_p(1-\beta_i(\beta_{i+1},0)) - 2c) \left[\prod_{k=p+1}^{i} (1-\lambda_k) \right] \right). \tag{9}$$

The first derivative of (9) w.r.t. β_{i+1} is $-\sum_{p=1}^{i-1} \left(2^{p-i}\lambda_p \frac{\partial \beta_i(\beta_{i+1},0)}{\partial \beta_{i+1}} \left[\prod_{k=p+1}^{i}(1-\lambda_k) \right] \right) < 0$. The second derivative of (9) w.r.t. β_{i+1} is $-\sum_{p=1}^{i-1} \left(2^{p-i}\lambda_p \frac{\partial^2 \beta_i(\beta_{i+1},0)}{\partial \beta^2_{i+1}} \left[\prod_{k=p+1}^{i}(1-\lambda_k) \right] \right) < 0$. Thus, (9) is strictly concave over $\beta_{i+1} \in (0, \beta^*_{i+1})$. Since the derivative of $LHS(i)$ w.r.t. β_{i+1} is constant and equal to $-\lambda_i < 0$ and since, by assumption, $LHS(i) > RHS(i)$ if $\chi_j(\beta) = 0$ for $j < i$ and $\beta < \beta_{i+1}$, there can be no more than one solution to (4) in period i. Holding χ_i "fixed" at a solution to (4) at period i, it follows that $LHS(i)$ falls more quickly with an increase in β_{i+1} than $RHS(i)$. To preserve equality, since $LHS(i)$ is independent of χ_i and $RHS(i)$ rises with χ_i, χ_i must decrease with β_{i+1}, strictly so for $\chi_i \in (0, 1)$. ■

Proof of Corollary 1: Clearly, the $RHS(i)$ and $LHS(i)$ of (4) are continuous and decreasing in β_{i+1}, and $LHS(i) > RHS(i)$ for $\beta_{i+1} \geq 1 - \frac{2c}{\lambda_i}$. Hence, a sufficient condition for the speculator to trade with positive probability at period i is that $RHS(i) > LHS(i)$ when evaluated at $\beta_{i+1} = 0$. This sufficient condition corresponds to $\lambda_i - 2c > \sum_{p=1}^{i-1} \left(\frac{\lambda_p - 2c}{2^{i-p}} \left[\prod_{k=p+1}^{i}(1-\lambda_k) \right] \right)$ for $i > 1$ and to $\lambda_i > 2c$ for $i = 1$. Given **A1**, if the sufficient condition holds at period i, the speculator will submit an order each period j, $j \leq i$. When $\lambda_i = \lambda \; \forall i$, the sufficient condition simplifies to $\lambda - 2c > (\lambda - 2c) \sum_{p=1}^{i-1} \left[\frac{1-\lambda}{2} \right]^p$, which is always satisfied if $\lambda - 2c > 0$. ■

Proof of Proposition 3: **Change in λ_i:** Interior values for the period i trading probabilities $\chi_i(\beta_{i+1})$ must solve (4). When λ_i decreases, $LHS(i)$ decreases and $(1 - \lambda_i)$ increases. In order to maintain equality, it follows that $V_{i-1}(\beta_i(\beta_{i+1}, 0))$ decreases and thus $\chi_i(\beta_{i+1})$ decreases.

Change in λ: The period i analog to (4) for the case where $\lambda_i = \lambda$ is

$$\lambda(1-\beta_{i+1}) - 2c = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1},0)). \tag{10}$$

Let $LHS^*(i)$ and $RHS^*(i)$ be the period i values of the left-hand side and the right-hand side, respectively, of (10). Part A of proposition 1 implies that: (a) a solution to (10) can occur only for

$\beta_{i+1} \in \left(0, 1 - \frac{2c}{\lambda}\right)$; and (b) $\chi_j(\beta) > 0 \; \forall j \forall \beta < 1 - \frac{2c}{\lambda}$. Hence, for interior values of $\chi_i(\beta_{i+1})$ it follows that $V_{i-1}(\beta_i(\beta_{i+1}, 0)) = \frac{(1-\lambda)}{2} V_{i-2}(\beta_i(\beta_{i+1}, 0)) + \frac{\lambda}{2}(1 - \beta_i(\beta_{i+1}, 0)) - c$.

Period 1: Recall that $\chi_1(\beta_2) = 1$ if $\beta_2 < 1 - \frac{2c}{\lambda}$ and $\chi_1(\beta_2) = 0$ if $\beta_2 > 1 - \frac{2c}{\lambda}$. Since $\frac{\partial}{\partial \lambda}\left[1 - \frac{2c}{\lambda}\right] = \frac{2c}{\lambda^2} > 0$, it follows that $\chi_1(\beta_2)$ is weakly increasing in λ.

Period $i > 1$: Define $a_i = \sum_{k=1}^{i-1} 2^{1-k}(1-\lambda)^k < 2$, for $i = 2, 3, ..., T$. Observe that
$\lambda \frac{\partial LHS^*(i)}{\partial \lambda} - a_i c = \lambda(1 - \beta_{i+1}) - a_i c > \lambda(1 - \beta_{i+1}) - 2c = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1}, 0))$,
and $\lambda \frac{\partial RHS^*(i)}{\partial \lambda} - a_i c = (1-\lambda)\left[\sum_{p=1}^{i-1}\left[\frac{(1-\lambda)}{2}\right]^{p-1}\left[\frac{\lambda}{2}(1 - \beta_i(\beta_{i+1}, 0)) - V_{i-1-p} - c\right]\right]$
$< (1-\lambda)\left[\sum_{p=1}^{i-1}\left[\frac{(1-\lambda)}{2}\right]^{p-1}\left[\frac{\lambda}{2}(1 - \beta_i(\beta_{i+1}, 0)) - c\right]\right] = (1-\lambda)V_{i-1}(\beta_i(\beta_{i+1}, 0))$.
It follows that $\frac{\partial LHS^*(i)}{\partial \lambda} > \frac{\partial RHS^*(i)}{\partial \lambda}$. To maintain the equality, $\chi_i(\beta_{i+1})$ must increase. Thus, $\chi_i(\beta_{i+1})$ is weakly increasing in λ. ■

Proof of Proposition 4 (Change in c): Interior values of the mixing probability χ_i are defined by (4). Differentiating $LHS(i)$ and $RHS(i)$ of (4) with respect to c: $\left.\frac{\partial RHS(i)}{\partial c}\right|_{\chi_i} = \left.\frac{\partial V_{i-1}}{\partial c}\right|_{\chi_i} \geq -1 > \frac{-2}{1-\lambda_i} = \frac{\partial LHS(i)}{\partial c}$. To restore equilibrium, $RHS(i)$ must decrease. Hence, χ_i must fall. ■

Proof of Proposition 5 (Change in δ): For a given χ_i, an increase in δ causes the market maker's belief about the probability of the good state to fall more rapidly in response to observing an aggregate order flow of zero $\left(\left.\frac{\partial \beta_i(\beta_{i+1},0)}{\partial \delta}\right|_{\chi_i} \leq 0\right)$. As δ increases, $\beta_i(\beta_{i+1}, 0)$ falls and $V_{i-1}(\beta_i(\beta_{i+1}, 0))$ increases. Hence, $RHS(i)$ of (4) rises with δ. Since $LHS(i)$ does not vary with δ, χ_i must fall to restore the mixing equilibrium condition given by (4).■

Lemma 3 *In equilibrium, the speculator never places a order of size two at period i, $i \geq 2$.*

The payoff from submitting an order of size two at period i, $i \geq 2$, is $\left(\sum_{\iota=i+1}^{T} X_\iota\right) - c$. When $\lambda_j > 0, j = 2, ..., i$, the speculator can realize a higher expected return of $\sum_{\iota=i+1}^{T} X_\iota - c\left[\Pi_{j=2}^{i}(1 - \lambda_j)\right]$ by deferring each period from period i to period 2 and, if his information has not yet been revealed, submit an order of size two at period 1. ■

Proof of Proposition 6: First observe that

$$\frac{\partial E[\pi_1(X_1|\mathbf{H}_2, \sum_{\iota=2}^T X_\iota)]}{\partial(\sum_{\iota=2}^T X_\iota)} = \begin{cases} 1 & \text{if } X_1 = 2 \\ 0.5 + 0.5\lambda_1 & \text{if } X_1 = 1 \\ \lambda_1 & \text{if } X_1 = 0 \end{cases}. \tag{11}$$

Since $1 \geq 0.5 + 0.5\lambda_1 \geq \lambda_1$ (strict when $\lambda_1 < 1$), it follows that the speculator is more likely to submit a larger order at period 1 as $(\sum_{\iota=2}^T X_\iota)$ increases. For period 2, Lemma 3 ensures that the speculator never submits an order of size two. Conditional on $\sum_{\iota=3}^T X_\iota$, the informed's expected return from a round lot order at period 2 is

$$submit\left(\sum_{\iota=3}^T X_\iota\right) = \frac{(1-\lambda_2)}{2} V_1\left(Y_2 = 1, \sum_{\iota=3}^T X_\iota + 1\right) + \frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1)\right) - c + \frac{(1+\lambda_2)}{2}\sum_{\iota=3}^T X_\iota,$$

and the expected return from deferring at period 2 is

$$defer\left(\sum_{\iota=3}^T X_\iota\right) = \frac{(1-\lambda_2)}{2}\left[V_1\left(Y_2 = 1, \sum_{\iota=3}^T X_\iota\right) + V_1\left(Y_2 = 0, \sum_{\iota=3}^T X_\iota\right)\right] + \lambda_2 \sum_{\iota=3}^T X_\iota,$$

where we suppress the order flow history, $\mathbf{H}_3$, and only report Y_2 to conserve space. If $submit(Q) - defer(Q) \geq submit(\tilde{Q}) - defer(\tilde{Q})$, for all $Q > \tilde{Q}$, then the relative value of submitting an order rises weakly with the size of the speculator's accumulated position. Rewriting this condition yields

$$\Big([V_1(1, Q+1) - V_1(1, \tilde{Q}+1)] - [V_1(0, Q) - V_1(0, \tilde{Q})]\Big) + \Big(Q - \tilde{Q} - [V_1(1, Q) - V_1(1, \tilde{Q})]\Big) \geq 0. \tag{12}$$

Equation (11) shows that the magnitude of the increase in the value of the speculator's information at period 1, due to an increase in $(\sum_{\iota=2}^T X_\iota)$, rises with X_1. It also ensures that for any given market maker beliefs, the speculator is more likely to submit a larger order at period 1 as $(\sum_{\iota=2}^T X_\iota)$ increases. These two observations imply that $V_1(1, Q+1) - V_1(1, \tilde{Q}+1) \geq V_1(1, Q) - V_1(1, \tilde{Q})$. To show that $Q - \tilde{Q} \geq V_1(0, Q) - V_1(0, \tilde{Q})$, we first observe that (11) implies that $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] \geq \text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, \tilde{Q})]$. Consider each possible case:

(1) If $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = \text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, \tilde{Q})]$, then

$$Q - \tilde{Q} \geq V_1(0, Q) - V_1(0, \tilde{Q}) = \begin{cases} \lambda_1(Q - \tilde{Q}) & \text{if } \text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 0 \\ 0.5(1+\lambda_1)(Q - \tilde{Q}) & \text{if } \text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 1 \\ Q - \tilde{Q} & \text{if } \text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 2 \end{cases}.$$

(2) If $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 2$ and $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, \tilde{Q})] = 1$, then

$$\tilde{Q} < \frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1, \mathbf{H}_3)\right) + \frac{(1+\lambda_1)}{2}\tilde{Q}.$$

Then $V_1(0,Q) - V_1(0,\tilde{Q}) = Q - \frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1, \mathbf{H}_3)\right) - \frac{(1+\lambda_1)}{2}\tilde{Q} < Q - \tilde{Q}$.

(3) If $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 1$ and $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, \tilde{Q})] = 0$, then

$$\frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1, \mathbf{H}_3)\right) - c + \frac{(1+\lambda)}{2}\tilde{Q} < \lambda\tilde{Q}$$

$$\frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1, \mathbf{H}_3)\right) - c + \frac{(1+\lambda)}{2}\tilde{Q} + \frac{(1+\lambda)}{2}(Q - \tilde{Q}) - \lambda\tilde{Q} < Q - \tilde{Q}$$

Then $V_1(0,Q) - V_1(0,\tilde{Q}) = \frac{\lambda_2}{2}\left(1 - P_2(X_2 = 1, \mathbf{H}_3)\right) - c + \frac{(1+\lambda)}{2}Q - \lambda\tilde{Q} < Q - \tilde{Q}$.

(4) Finally, if $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, Q)] = 2$ and $\text{argmax}_{X_1} E[\pi_1(X_1|\mathbf{H}_2, \tilde{Q})] = 0$, then $\tilde{Q} - c - \lambda\tilde{Q} < 0$. This implies directly that $V_1(0,Q) - V_1(0,\tilde{Q}) = Q - c - \lambda\tilde{Q} < Q - \tilde{Q}$.

These observations ensure that condition (12) is satisfied, and hence, $\chi_2(X_2 = 1|\sum_{\iota=3}^{T} X_\iota = Q, \cdot) \geq \chi_2(X_2 = 1|\sum_{\iota=3}^{T} X_\iota = \tilde{Q}, \cdot), \forall Q > \tilde{Q}$.■

Lemma 4 *In equilibrium, the speculator never submits an order for more than one contract at period 1.*

$E\left[\pi_1(1, \mathbf{Y}_2, \mathbf{X}_2) - \pi_1(2, \mathbf{Y}_2, \mathbf{X}_2)\right] = (1-\Sigma)(1-\frac{1}{2}\beta_1(1,\mathbf{Y}_2)-\frac{1}{2}\beta_1(2,\mathbf{Y}_2))+\Sigma(\frac{1}{2}-\frac{1}{2}\beta_1(1,\mathbf{Y}_2))-(1-\Sigma)(1-\beta_1(2,\mathbf{Y}_2)) = \frac{1}{2}(1-\Sigma)(\beta_1(2,\mathbf{Y}_2)-\beta_1(1,\mathbf{Y}_2))+\frac{1}{2}\Sigma(1-\beta_1(1,\mathbf{Y}_2)) > 0$ where $\Sigma = \Sigma(\mathbf{Y}_2, \mathbf{X}_2)$. The result then follows from $\beta_1(2,\mathbf{Y}_2) > \beta_1(1,\mathbf{Y}_2)$ and $1 > \beta_1(1,\mathbf{Y}_2)$. ■

Lemma 5 *In equilibrium, the speculator never submits an order involving two contracts at period 2.*

More aggressive period 2 strategies are most profitable when there are no continuation profits ($\lambda_2 = 1$). Even for this most favorable parameter value, each of the 3 possible strategies involving 2 contracts is still less profitable than at least one strategy involving 1 contract:

Case 1: $E\left[\pi_2(0,2) - \pi_2(0,1)\right] = 1 + \frac{1}{8}\left[-10 + (3-2\xi)\left(\beta_2(1,1) - \beta_2(0,2)\right) + 2\beta_2(0,1)\right]$. This is negative iff $(3-2\xi)\left(\beta_2(1,1) - \beta_2(0,2)\right) < 2\left(1 - \beta_2(0,1)\right)$. The desired result follows from: $\frac{1}{2}(1+2\delta(\xi\rho_S+(1-\xi)\rho_L)) < \frac{5}{2}$, which implies that $3(1+2\delta(\xi\rho_S+(1-\xi)\rho_L)) < \frac{5}{2}(1+2\delta(\xi\rho_S +$

$(1-\xi)\rho_L)) + \frac{5}{2}$. Rearranging, we have $3\beta_2(1,1) < \frac{5}{2}$. Thus, given that $\beta_2(0,1) \leq \beta_2(0,2)$ and $\beta_2(0,2) > \frac{1}{2}$, it follows that: $3\beta_2(1,1) + 2\beta_2(0,1) < 2 + 3\beta_2(0,2) \Rightarrow 3\left(\beta_2(1,1) - \beta_2(0,2)\right) < 2\left(1 - \beta_2(0,1)\right) \Rightarrow (3-2\xi)\left(\beta_2(1,1) - \beta_2(0,2)\right) < 2\left(1 - \beta_2(0,1)\right)$.

Case 2: $E\left[\pi_2\left(2,0\right) - \pi_2\left(1,0\right)\right] = 1 + \frac{1}{8}\left[-10 + (1+2\xi)\left(\beta_2(1,1) - \beta_2(2,0)\right) + 2\beta_2(1,0)\right]$. This is negative iff $(1+2\xi)\left(\beta_2(1,1) - \beta_2(2,0)\right) < 2\left(1 - \beta_2(1,0)\right)$. From case 1, we know that $3\beta_2(1,1) < \frac{5}{2}$. Since $\beta_2(1,0) \leq \beta_2(2,0)$ and $\beta_2(2,0) > \frac{1}{2}$, the desired result follows from: $3\beta_2(1,1) + 2\beta_2(1,0) < 2 + 3\beta_2(2,0) \Rightarrow 3\left(\beta_2(1,1) - \beta_2(2,0)\right) < 2(1 - \beta_2(1,0)) \Rightarrow (1 + 2\xi)\left(\beta_2(1,1) - \beta_2(2,0)\right) < 2(1 - \beta_2(1,0))$.

Case 3: $E\left[\pi_2(1,1) - \pi_2(1,0)\right] = (1-c) + \frac{1}{8}\left[-10 + (3-2\xi)(\beta_2(2,0) - \beta_2(1,1)) + 2\beta_2(1,0)\right]$. This is negative iff $(3-2\xi)\left(\beta_2(2,0) - \beta_2(1,1)\right) < 2\left(1 - \beta_2(1,0)\right)$. This will always be true for $\xi > \frac{1}{2}$. For $\xi \leq \frac{1}{2}$, the result follows from: $(3-2\xi)\delta\rho_S + (1-\xi)(1-\delta+\delta\rho_N) + 5\xi < 5 \Rightarrow 3((3-2\xi)\delta\rho_S + (1-\xi)(1-\delta+\delta\rho_N)) < \frac{5}{2}((3-2\xi)\delta\rho_S + (1-\xi)(1-\delta+\delta\rho_N)) + \frac{5}{2}(1-\xi)$. Rearranging, we have $\beta_2(2,0) < \frac{5}{2}$. Since $\beta_2(1,1) \geq \beta_2(0,1)$ and $\beta_2(1,1) > \frac{1}{2}$, it follows that: $3\beta_2(2,0) + 2\beta_2(0,1) < 2 + 3\beta_2(1,1) \Rightarrow 3(\beta_2(2,0) - \beta_2(1,1)) < 2 - 2\beta_2(1,0) \Rightarrow (3 - 2\xi)\left(\beta_2(2,0) - \beta_2(1,1)\right) < 2\left(1 - \beta_2(1,0)\right)$.■

References

Back, K. (1993). Asymmetric Information and Options. Review of Financial Studies, 6, 435–472.

Back, K., Cao, H. H., & Willard, G. A. (2000). Imperfect competition among informed traders. Journal of Finance, 55, 2117–2155.

Biais, B., & Hillion, P. (1994). Insider and Liquidity Trading in Stock and Options Markets. Review of Financial Studies, 7, 743–780.

Brennan, M. J., & Crew, N. I. (1997). Hedging long maturity commodity commitments with short-dated futures contracts. In M. A. H. Dempster & S. R. Pliska (eds), Mathematics of Derivative Securities (pp. 165–187). Cambridge University Press.

Castelino, M., & Francis, J. C. (1982). Basis speculation in commodity futures: The maturity effect. The Journal of Futures Markets, 2, 195–206.

Chakravarty, S., Guyen, H., & Mayhew, S. (2004). Informed trading in stock and option markets. Journal of Finance, 59, 1235–1257.

Dow, J., & Gorton, G. (1994). Arbitrage chains. Journal of Finance, 49, 819–849.

Easley, D., O'Hara, M., & Srinivas, P. S. (1998). Option Volume and Stock Prices: Evidence on Where Informed Traders Trade. Journal of Finance, 53, 431–465.

Fleming, M., & Sarkar, A. (1999). Liquidity in U.S. treasury spot and futures markets. In Report No. 11 Market Liquidity: Research Findings and Selected Policy Implications. Bank for International Settlements, Committee on the Global Financial System.

Foster, F. D., & Viswanathan, S. (1996). Strategic trading when agents forecast the forecasts of others. Journal of Finance, 51, 1437–1478.

Gardner, B.L. (1989). Rollover hedging and missing long-term futures markets. American Journal of Agricultural Economics, 71, 311–318.

Goldstein, I., & Guembel, A. (2004). Manipulation and the allocational role of prices. Working paper, Duke University.

Han, L., Kling, J., & Sell, C. (1999). Foreign Exchange Futures Volatility: Day-of-the-Week, Intraday, and Maturity Patterns in the Presence of Macroeconomic Announcements. The Journal of Futures Markets, 19, 665–693.

Herbert, J. H. (1995). Trading volume, maturity, and natural gas futures price volatility. Energy Economics, 17, 293–299.

Holden, C. W., & Subrahmanyam, A. (1992). Long-lived private information and imperfect competition. Journal of Finance, 47, 247–270.

Hong, H. (2000). A model of returns and trading in futures markets. Journal of Finance, 55, 959–988.

John, K., Koticha, A., Narayanan, R., & Subrahmanyam, M. (2001). Margin Requirements, Informed Trade in Derivatives and Price Dynamics. Working Paper FIN-01-038, New York University Stern School of Business, March.

Kyle, A. S. (1985). Continuous auctions and insider trade. Econometrica, 53, 1315–1335.

Lence, S. H., & Hayenga, M. L. (2001). On the pitfalls of multi-year rollover hedges: The case of hedge-to-arrive contracts. American Journal of Agricultural Economics, 83, 107–119.

Lestikow, D. (1989). Announcements and Futures Price Variability. The Journal of Futures Markets, 9, 477–486.

Lien, D., & Wang, Y. (2004). Hedging long-term commodity risk: A comment. The Journal of Futures Markets, 29, 1093–1099.

McCabe, G. M., & Franckle, C.T. (1983). The effectiveness of rolling the hedge forward in the Treasury Bill futures market. Financial Management, 12, 21–29.

Movassagh N., & Modjtahedi, B. (2005). Bias and backwardation in natural gas futures prices. The Journal of Futures Markets, 25, 281–308.

Neuberger, A. (1999). Hedging long-term exposures with multiple short-term futures contracts. Review of Financial Studies, 12, 429–459.

Routledge, B. R., Seppi, D. J., & Spatt, C. S. (2000). Equilibrium forward curves for commodities. Journal of Finance, 55, 1297–1338.

Rutledge, D. J. S. (1976). A Note on the Variability of Futures Prices. Review of Economics and Statistics, 58, 118–120.

Samuelson, P. A. (1965). Proof that properly anticipated prices fluctuate randomly. Industrial Management Review, 6, 41–50.

Samuelson, P. A. (1976). Is real-world price a tale told by the idiot of chance?. Review of Economics and Statistics, 58, 120–123.

Seppi, D. J. (1990). Equilibrium block trading and asymmetric information. Journal of Finance, 45, 73–94.

Treynor, J. L., & Ferguson, R. (1985). In Defence of Technical Analysis. Journal of Finance, 40, 757–773.

Veld-Merkoulova, Y. V., & De Roon, F.A. (2003). Hedging long-term commodity risk. The Journal of Futures Markets, 23, 109–133.

Walls, W. D. (1999). Volatility, volume and maturity in electricity futures. Applied Financial Economics, 9, 283–287.

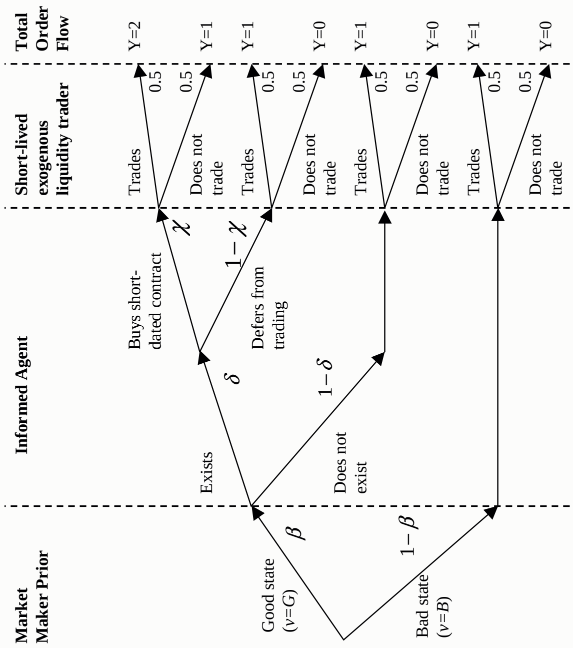


Figure 1: Period trading strategies and total order flow based on the basic model outlined in section 2. The probabilities associated with each strategy reflect those used by the market maker to update her beliefs.

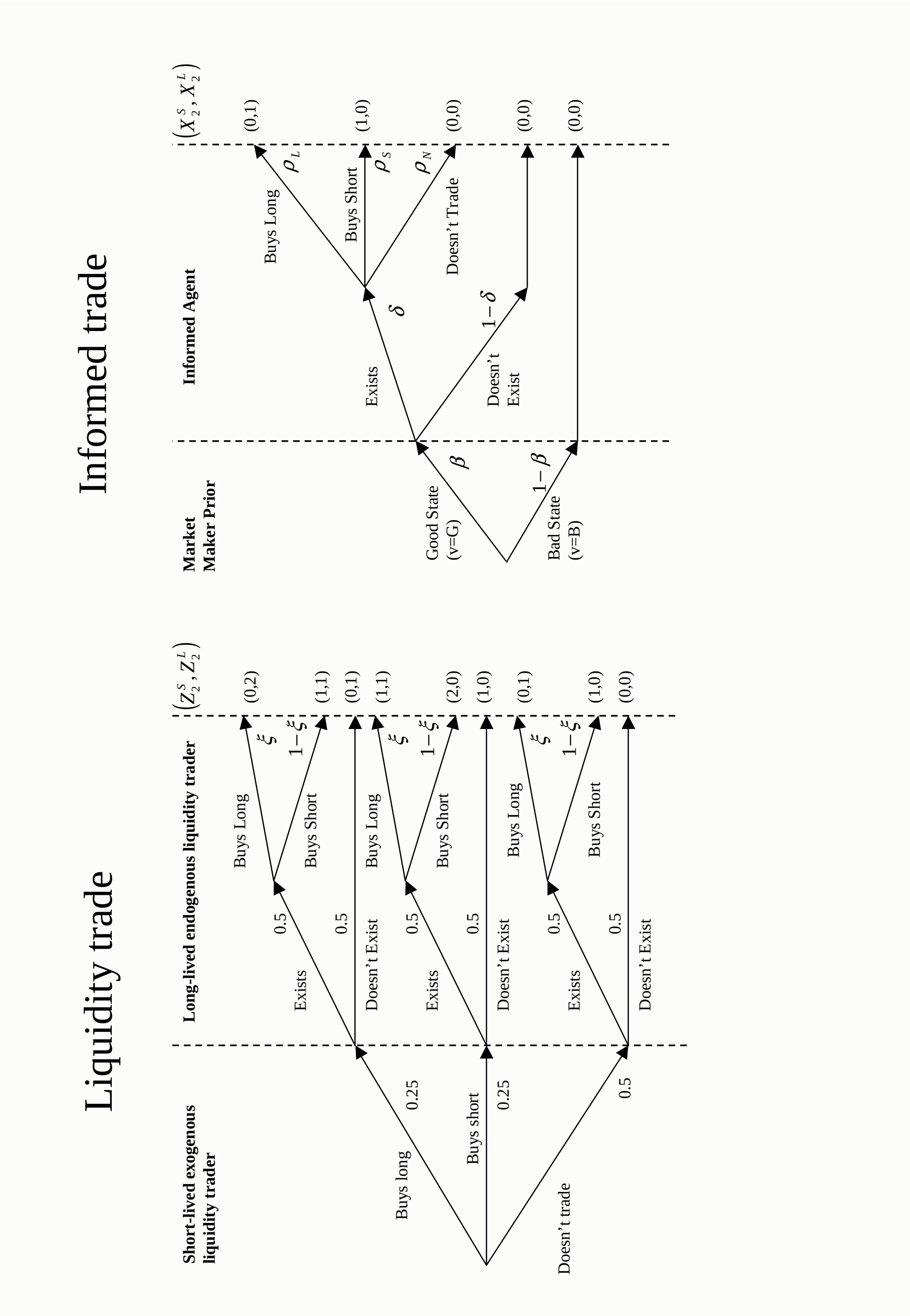


Figure 2: Period 2 trading strategies, total informed order flow and total liquidity order flow based on the model outlined in section 4.2. The probabilities associated with each strategy reflect those used by the market maker to update her beliefs. At period 2, the speculator defers from trade with probability $\rho_N = \Pr\{X_2^S = 0, X_2^L = 0\}$; buys a long-dated contract with probability $\rho_L = \Pr\{X_2^S = 0, X_2^L = 1\}$; and buys a short-dated contract with probability $\rho_S = \Pr\{X_2^S = 1, X_2^L = 0\}$.

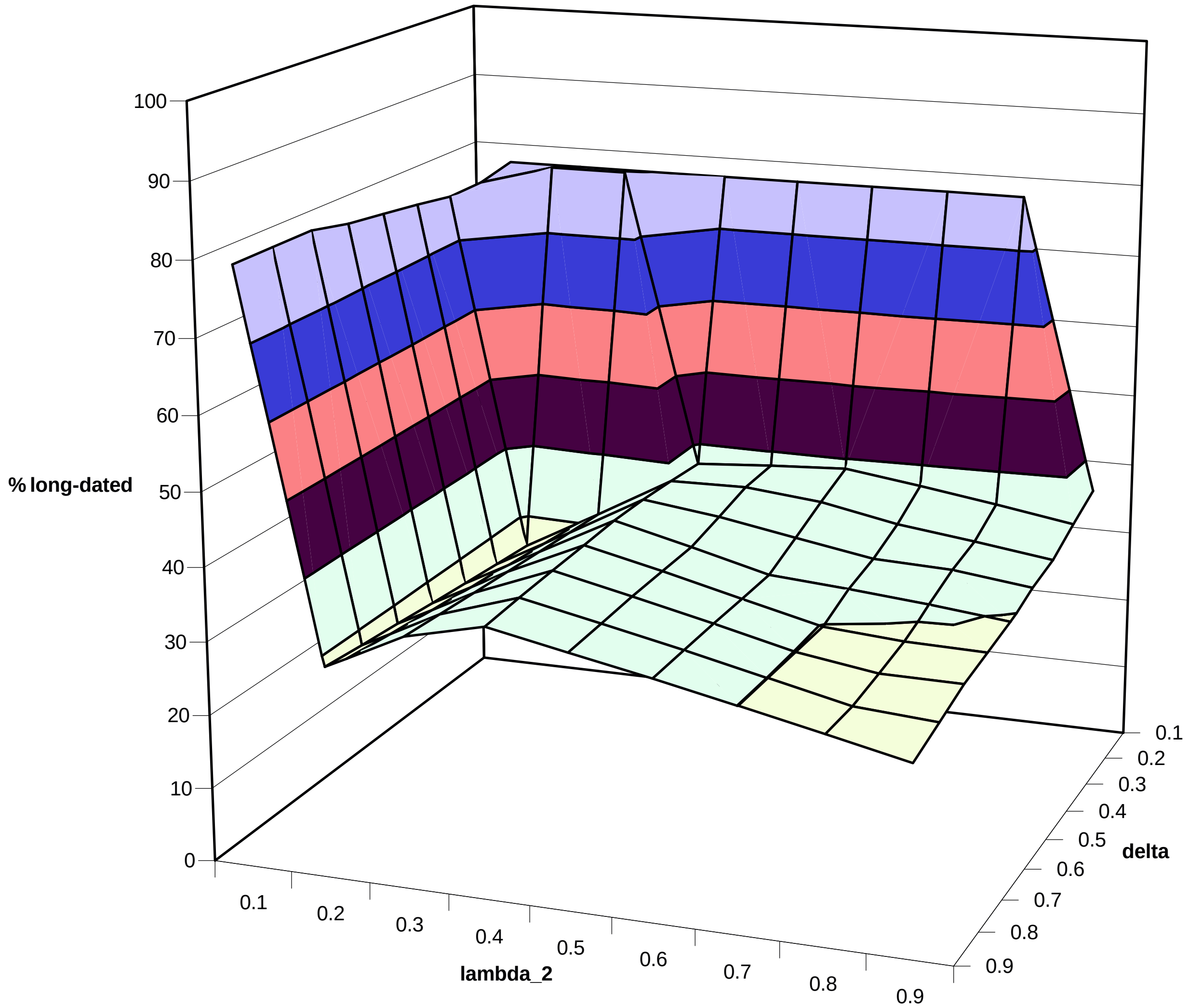


Figure 3: Surface diagram of the percentage of trade in long-dated contracts given fixed transaction costs equal to $c = 0.05$ for different values of δ (the probability a speculator has private information) and λ_2 (the probability of an information leakage in period 2). The percentage of trade in long-dated contracts is calculated as $(100(0.25 + 0.5\xi + \delta \Pr\{X_2^S = 0, X_2^L = 1\})/(1 + \delta(\Pr\{X_2^S = 0, X_2^L = 1\} + \Pr\{X_2^S = 1, X_2^L = 0\}))$.

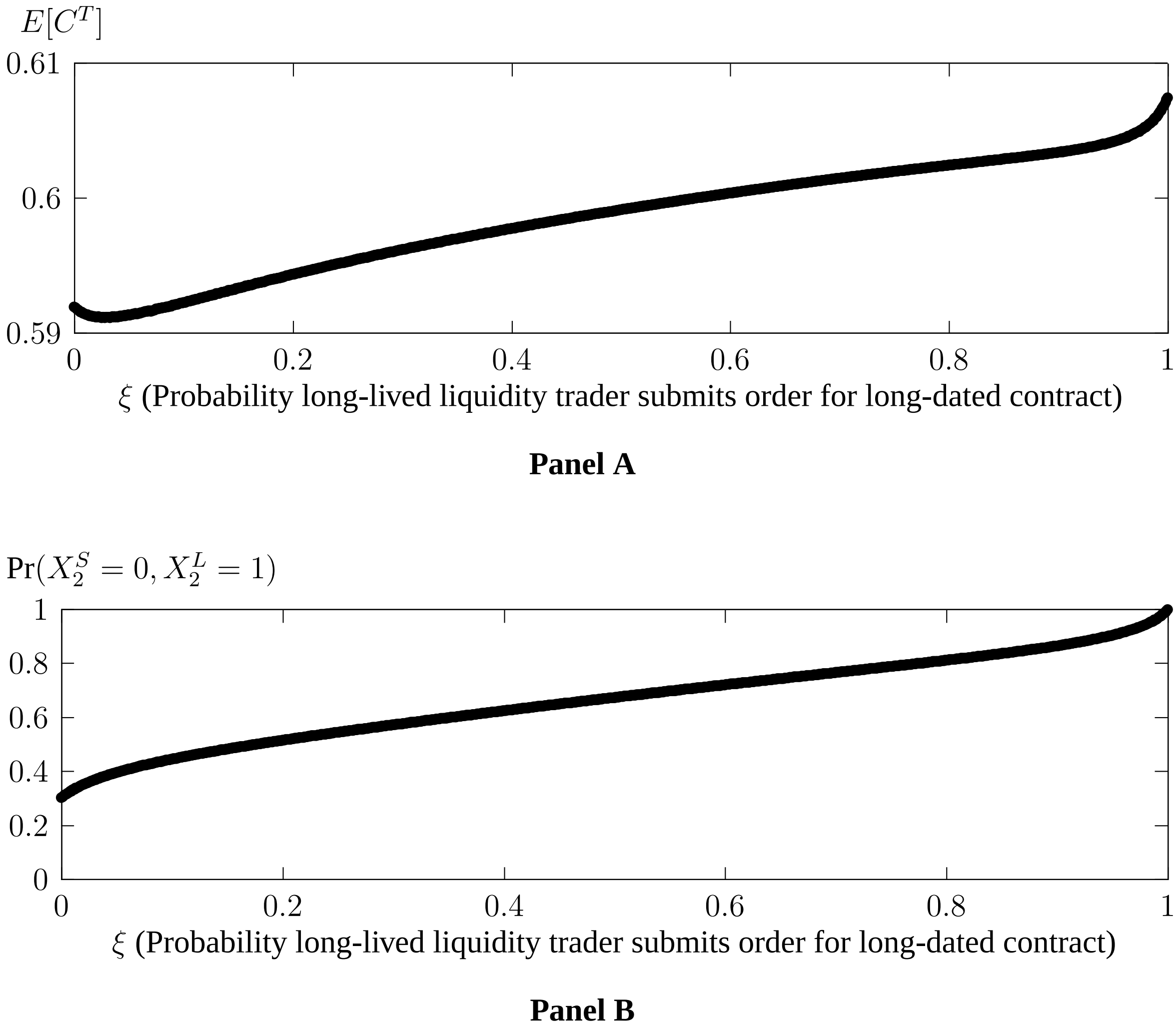
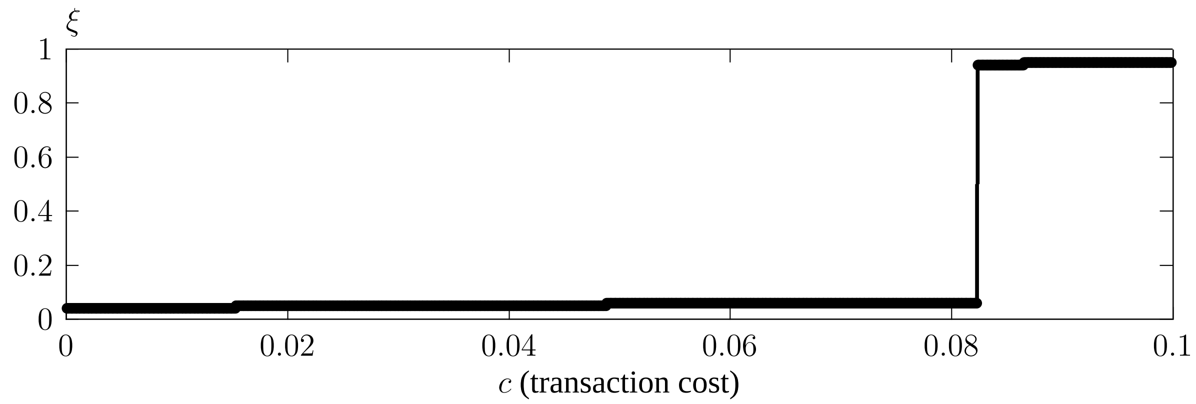
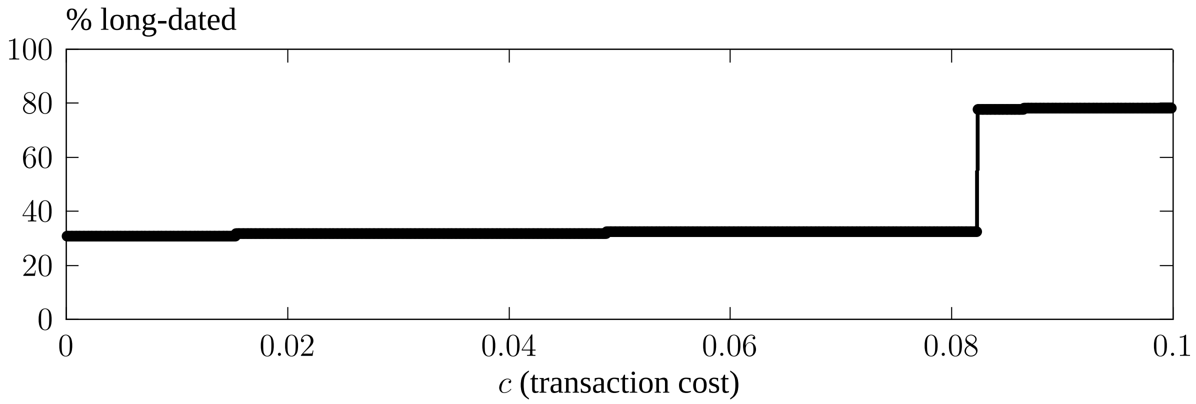


Figure 4: Panel A shows the long-lived liquidity trader's expected total cost, $E[C^T]$, as a function of his mixing probability ξ. Panel B shows the corresponding informed agent's probability of submitting an order to the long-dated contract market ($\Pr\{X_2^S = 0, X_2^L = 1\}$) as a "best response" to the long-lived liquidity trader's mixing probability ξ. The long-lived liquidity trader always purchases the short-dated contract when $\xi = 0$ and always purchases the long-dated contract when $\xi = 1$. Parameter values: $\delta = .5$, $c = .01$, and $\lambda_2 = .8$. For these parameters, it is always optimal for the speculator to trade at period 2.



Panel A



Panel B

Figure 5: **Liquidity trader's preference for "pooling equilibrium".** Panel A shows the long-lived liquidity trader's mixing probability ξ as a function of the fixed transaction cost c. Panel B shows the percentage of total order flow in long-dated contracts as a function of the fixed transaction cost c. The long-lived liquidity trader always purchases the short-dated contract when $\xi = 0$ and always purchases the long-dated contract when $\xi = 1$. The figure is based on the parameter values: $\lambda_2 = 0.8$ (the unconditional probability of exogenous information revelation at period 2 is 80%, "High") and $\delta = 0.4$ (the probability that a speculator has information about the risky asset is 40%).

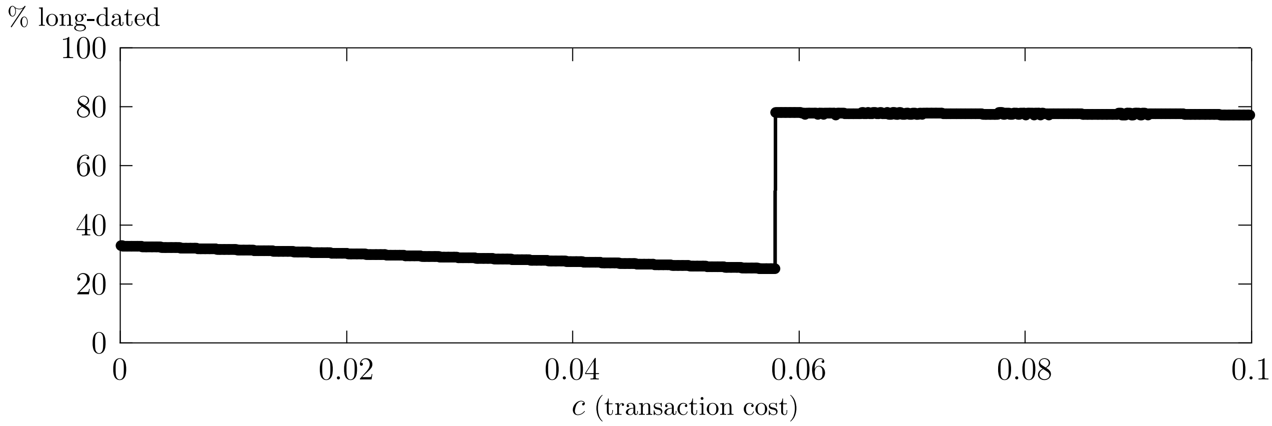


Figure 6: **Low unconditional probability of exogenous information revelation at period 2 ($\lambda_2 = 0.2$).** The figure shows the percentage of total order flow in long-dated contracts as a function of the fixed transaction cost c. The figure is based on $\delta = 0.4$ (the probability that a speculator has information about the risky asset is 40%).