Fund of Hedge Funds Portfolio Selection: A Multiple-Objective Approach

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This paper develops a technique for fund of hedge funds to allocate capital across different hedge fund strategies and traditional asset classes. Our adaptation of the Polynomial Goal Programming (PGP) optimization method incorporates investor preferences for higher return moments, such as skewness and kurtosis, and provides computational advantages over rival methods. We show how optimal allocations depend on the interaction between strategies, as measured by covariance, co-skewness and co-kurtosis. We also demonstrate the importance of constructing "like for like" representative portfolios that reflect the investment opportunities available to different sized funds. Our empirical results reveal the importance of equity market neutral funds as volatility and kurtosis reducers, and of global macro funds as portfolio skewness enhancers.

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As hedge funds continue to become more and more popular with investors, the amount of assets under their management has steadily grown, from around $40 billion in 1990 to an estimated $1,182 billion in 2006. Most investors do not invest in individual hedge funds directly, but invest in so-called funds of hedge funds (FoHF) instead. In return for a typically not-insignificant fee, FoHF (claim to) take care of the many unavoidable, time-consuming and complex issues that come with investing in a highly opaque asset class such as hedge funds. Although FoHF have been around for quite some time, it is still unclear how FoHF should optimally allocate capital across various hedge fund strategies.[1] In this paper, we show how a simple allocation technique based on Polynomial Goal Programming (PGP) is particularly well-suited to dealing with the complex return distributions of hedge funds and their practical institutional constraints.

Amin and Kat [2003b], Anson [2002], and others, show that hedge fund returns are substantially more complex than common stock and bond returns. Not only do hedge fund return distributions tend to exhibit significant skewness and kurtosis, they also tend to display significant co-skewness with the returns on other hedge funds as well as equity. As a result, standard mean-variance portfolio theory (as well as performance measures based on it, such as the Sharpe ratio) is inadequate when dealing with portfolios of (or including) hedge funds — a more extensive model is required.[2]

Here, we construct a PGP optimization model that is able to balance multiple conflicting and competing hedge fund allocation objectives: maximizing expected return while simultaneously minimizing return variance, maximizing skewness and minimizing kurtosis. We show how changes in investor preferences lead to different asset allocation across hedge fund strategies and across asset classes (hedge funds, stocks, and bonds). The PGP model provides guidance on how much capital, if any, should be allocated to each hedge fund strategy. In this way, a fund of funds can incorporate the investment goals of its target investors to

[1]MeesPierson's Leveraged Capital Holdings, one of the first multi-manager FoHF, was introduced in 1969.

[2]Of course, this implicitly assumes that investors' utility functions are of higher order than quadratic. See Jean [1971] and Scott and Horvath [1980] for details.

help determine whether it should hold a wide cross-section of strategies (as the majority of FoHF do), or instead should focus its expertise on a few strategies or a single strategy.

We proceed as follows. The next section formulates optimal hedge fund portfolio selection within a 4-moment framework as a multiple objective problem. Section 2 describes the data. Section 3 provides illustrative empirical results. Section 4 concludes. The appendix outlines the procedure used for unsmoothing the raw hedge fund return data.

1 Portfolio Selection in a 4-Moment Framework

The PGP approach was first used in finance by Tayi and Leonard [1988] to facilitate bank balance sheet management. It has subsequently been used by Lai [1991], Chunhachinda, et al. [1997], Sun and Yan [2003], and Prakash et al. [2003] to solve portfolio selection problems involving a significant degree of skewness. Here, we adapt the basic PGP approach to the context of FoHF portfolio selection. In order to incorporate more information about the non-normality of returns, we augment the dimensionality of the PGP portfolio selection problem from mean-variance-skewness to mean-variance-skewness-kurtosis. Later, we describe how we construct representative portfolios for each hedge fund strategy.

Consider an environment with $n+1$ assets. Each of the assets $1, 2, \ldots, n$ is a portfolio of hedge funds selected in a manner described below to represent a typical portfolio of funds drawn from each of the n hedge fund strategy classifications. Each strategy portfolio has a random return $\tilde{R}_i$. We impose a no short-sale requirement: negative positions in the portfolios of hedge funds are not allowed. Asset $n+1$ is the risk-free asset with rate of return r for both borrowing and lending.

Let x_i denote the percentage of wealth invested in the ith asset and let $\mathbf{X} = (x_1, x_2, \cdots, x_n)^\top$. Corresponding to $\tilde{\mathbf{R}} = (\tilde{R}_1, \tilde{R}_2, \ldots, \tilde{R}_n)^\top$, is a positive definite $n \times n$ variance-covariance matrix $\mathbf{V}$. The percentage invested in the risk-free asset is determined by $x_{n+1} = 1 - \mathbf{I}^\top\mathbf{X}$, where $\mathbf{I}$ is a $n \times 1$ identity vector. Since the portfolio decision depends on the *relative* percentage invested in each asset, the portfolio choice $\mathbf{X}$ can be rescaled and restricted on the unit variance space (i.e., $\{\mathbf{X} \mid \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1\}$). Then, the portfolio selection problem may be stated as the following multiple objective programming problem:

$$\begin{aligned}
\text{Maximize} \quad & Z_1 = E\left[\mathbf{X}^\top \tilde{\mathbf{R}}\right] + x_{n+1} r, && (1)\\
\text{maximize} \quad & Z_3 = E\left[\mathbf{X}^\top \left(\tilde{\mathbf{R}} - E[\tilde{\mathbf{R}}]\right)\right]^3, && (2)\\
\text{minimize} \quad & Z_4 = E\left[\mathbf{X}^\top \left(\tilde{\mathbf{R}} - E[\tilde{\mathbf{R}}]\right)\right]^4, && (3)\\
\text{subject to} \quad & \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1; \qquad \mathbf{X} \geq 0; \qquad x_{n+1} = 1 - \mathbf{I}^\top \mathbf{X}. && (4)
\end{aligned}$$

where portfolio expected return is Z_1, skewness is Z_3, and kurtosis is Z_4.

Given an investor's preferences among objectives, a PGP can be expressed instead as:

$$\begin{aligned}
\text{Minimize} \quad & Z = (1 + d_1)^\alpha + (1 + d_3)^\beta + (1 + d_4)^\gamma, && (5)\\
\text{subject to} \quad & E\left[\mathbf{X}^\top \tilde{\mathbf{R}}\right] + x_{n+1} r + d_1 = Z_1^*, && (6)\\
& E\left[\mathbf{X}^\top \left(\tilde{\mathbf{R}} - E[\tilde{\mathbf{R}}]\right)\right]^3 + d_3 = Z_3^*, && (7)\\
& -E\left[\mathbf{X}^\top \left(\tilde{\mathbf{R}} - E[\tilde{\mathbf{R}}]\right)\right]^4 + d_4 = -Z_4^*, && (8)\\
& d_1,\ d_3,\ d_4 \geq 0, && (9)\\
& \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1; \qquad \mathbf{X} \geq 0; \qquad x_{n+1} = 1 - \mathbf{I}^\top \mathbf{X}. && (10)
\end{aligned}$$

where $Z_1^* = \text{Max}\{Z_1 \mid \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1\}$ is the mean return for the optimal mean-variance portfolio with unit variance, $Z_3^* = \text{Max}\{Z_3 \mid \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1\}$ is the skewness value of the optimal skewness-variance portfolio with unit variance, and $Z_4^* = \text{Min}\{Z_4 \mid \mathbf{X}^\top \mathbf{V} \mathbf{X} = 1\}$ is the kurtosis value of the optimal kurtosis-variance portfolio with unit variance; and where α, β and γ are the nonnegative investor-specific parameters representing the investor's subjective degree of preferences on the mean, skewness and kurtosis of the portfolio return. The specification of our objective function in (5) ensures that it is monotonically increasing in d_1, d_3, and d_4 for all possible values.

In summary, solving the multiple objective PGP problem involves a two-step procedure. First, the optimal values for Z_1^*, Z_3^* and Z_4^*, expected return, skewness and kurtosis, respectively, are each obtained within a unit variance two space framework. Subsequently, these values are substituted into the conditions (6)–(8), and the minimum value of (5) is found for a given set of investor preferences $\{\alpha, \beta, \gamma\}$ within the four-moment framework.

All the optimal portfolios obtained above are composed of risky assets (hedge fund strategy portfolios) and the risk-free asset, in order to ensure the uniqueness of each optimal portfolio. To capture an investor that is fully invested in hedge funds, we rescale the portfolio $\mathbf{X}$ such that the total investment is one (i.e. such that $x_{n+1} = 0$). Let $y_i = x_i/(x_1+x_2+\ldots+x_n)$ be the percentage invested in the ith asset in the optimal portfolio $\mathbf{Y}$. In the context of FoHF portfolios, y_i is the capital weight allocated to each hedge fund strategy in the optimal hedge fund portfolio.

When we investigate the asset allocation strategies for portfolios of stocks, bonds and hedge funds, there are $n+3$ assets in the world: n representative portfolios, one for each hedge fund strategy; the S&P500 index, representing stocks; the Salomon Brothers 7 Year Government Bond index (SALGVT7), representing bonds; and the risk-free asset. A no short-sale restriction is imposed for hedge fund portfolios only. Negative positions in stocks and/or bonds are allowed for.

Our PGP framework can be thought about in economic terms. Investors' utility will be augmented by a positive first moment (expected return), positive third moment (skewness) and negative fourth moment (kurtosis). The investor preference parameters α, β, and γ are directly associated with the marginal rate of substitution, which measures the desirability of forgoing one objective in order to gain another (conflicting) objective. For example, the marginal rate of substitution between expected return and skewness is given by $\frac{\partial Z/\partial d_1}{\partial Z/\partial d_3} = \frac{\alpha(1+d_1)^{\alpha-1}}{\beta(1+d_3)^{\beta-1}}$, and the marginal rate of substitution between expected return and kurtosis is given by $\frac{\partial Z/\partial d_1}{\partial Z/\partial d_4} = \frac{\alpha(1+d_1)^{\alpha-1}}{\gamma(1+d_4)^{\gamma-1}}$.

Thus, our approach allows users a simple, transparent method to specify their heterogeneous preferences for higher moments. This contrasts with standard portfolio optimization based on a specific utility function.[3] Recall that even the standard mean-variance utility function of Markowitz [1959] and Sharpe [1970] may be viewed as an approximation to the more basic von Neumann-Morgenstern utility function and more particularly to the isoelastic family of utility functions.[4] Unfortunately, these functions do not provide an exact preference ordering for risky portfolios using the first three (or higher) moments of portfolio returns. To isolate the impact of each moment, these non-polynomials are typically expanded using a Taylor series approximation. From an academic perspective, this is easily accommodated.[5] In practice, however, it is difficult for investors of hedge funds to describe their "utility function." Cremers, Kritzman, and Page [2005] use a full-scale optimization approach to show that different specifications of investor preferences (power utility, bilinear utility, and S-shaped value functions) imply considerable differences in the effect of higher return moments on optimal hedge fund allocations. Our simple approach largely mitigates these difficulties.

It is also important to highlight the advantages of using our approach over using a 1-stage linear objective function, such as:

$$\max \qquad E[R_p] - \lambda_1 \text{Variance}(R_p) + \lambda_2 \text{Skewness}(R_p) - \lambda_3 \text{Kurtosis}(R_p) \tag{11}$$

This linear objective function requires portfolio weights to be optimized over four dimensions simultaneously — potentially a very difficult computational problem. Worse yet, because skewness and kurtosis are the third and fourth return moments scaled by variance, each of the terms in the objective function interact with each other in complex ways, making trade-offs difficult to interpret. Thus, as the possible asset space increases in size, it becomes

[3]Examples in the context of hedge funds include Hagelin and Pramborg [2004] who develop a discrete-time dynamic investment model based on an investor with a power utility function and Barés et al. [2002] who examine the impact of hedge fund survival uncertainty on optimal allocations in an expected utility framework. In a more general asset return context, Harvey, et al. [2004] considers utility based portfolio optimization using a new Bayesian decision theoretic framework which incorporates higher moments and estimation error.

[4]Common choices are logarithmic, power, and negative exponential utility functions. These functions satisfy the desirable properties: (a) nonsatiety with respect to wealth, (b) risk aversion, and (c) risk assets are not inferior goods. See Grauer [2004] for more details.

[5]It is worth noting, however, that some of these utility-based approaches do not guarantee the existence of an optimal solution.

increasingly likely that a numerical solver will solve for a **local** maximum (or minimum) rather than the **global** maximum of (11).

In contrast, our PGP approach improves computational tractability by conducting the optimization in two stages. In the first stage, optimization is conducted in 2-dimensional space, thereby ensuring that a numerical solver always locates the global maximum (minimum) value of each moment. Then, in the second stage, optimization is conducted relative to these known targets (an easier computational problem). Investors can specify their preferences relative to these targets, allowing them more control and greater insights about the potential trade-offs. Throughout this process we set variance equal to one (which can later be re-scaled) — in effect, this means our optimization uses unscaled return moments, rather than skewness and kurtosis. This further improves computational tractability and also leads to better investment choices (see Bulhart and Klein [2005] for compelling evidence of this).

A natural question to ask is how our PGP approach performs out-of-sample relative to other approaches. Some evidence is provided by Anson, Ho, and Silberstein [2005], who use our PGP optimization approach to the CalPERS' hedge fund portfolio. They run the optimization process quarterly, and impose a 3-month lag between the optimization and the implementation (due to quarterly liquidity constraints). They find that out-of-sample performance of the optimized portfolios is better than mean-variance portfolios and that increasing investor preference parameters for the third and fourth return moment improves these moments out-of-sample. While this evidence is certainly encouraging, we caution readers that higher return moments can be driven by rare outliers, and therefore a more detailed out-of-sample analysis would require a longer history of hedge fund data than currently available.

Finally, before proceeding to our empirical results, we mention that another advantage of the PGP approach is that it can be adapted to embed other investor goals. For instance, based on an earlier version of this paper, at least one hedge fund has already adapted the PGP optimization function to incorporate a value-at-risk (VaR) measure. Thus, the method could incorporate features of other hedge fund allocation techniques, such as the mean-modified value-at-risk optimization procedure proposed by Favre and Galeano [2002].[6]

2 Strategy Classification and Data

Hedge fund investment strategies tend to be quite different from the strategies followed by traditional money managers. In principle every fund follows its own proprietary strategy, which means that hedge funds are a very heterogeneous group. It is, however, customary to ask hedge fund managers to classify themselves into one of a number of different strategy groups depending on the main type of strategy followed. We concentrate on seven main classes of funds. The numbers in square brackets indicate the estimated market share of each strategy group in terms of assets under management based on the June 2002 TASS asset flows report:

Long/Short Equity [43%]: Funds that simultaneously invest on both the long and the short side of the equity market. Unlike equity market neutral funds (see below), the portfolio may not always have zero market risk. Most funds have a long bias.

Equity Market Neutral [7%]: Funds that simultaneously take long and short positions of the same size within the same market, i.e. portfolios are designed to have zero market risk. Leverage is often applied to enhance returns.

Convertible Arbitrage [9%]: Funds that buy undervalued convertible securities, while hedging (most of) the intrinsic risks.

Distressed Securities [11%]: Funds that trade the securities of companies in reorganization and/or bankruptcy, ranging from senior secured debt to common stock.

Merger Arbitrage [8%]: Funds that trade the stocks of companies involved in a merger

[6]Other important papers in the hedge fund allocation literature include: Lamm [2003]'s Cornish-Fisher expansion, Terhaar et al. [2003]'s factor model, Alexander and Dimitriu [2004]'s statistical factor model approach, Cvitanić et al. [2003]'s manager ability uncertainty framework, Amec and Martellini [2002]'s out-of-sample model, and Popova et al. [2006]'s benchmark over/under performance construction.

or acquisition, buying the stocks of the company being acquired while shorting the stocks of its acquirer.

Global Macro [9%]: Funds that aim to profit from major economic trends and events in the global economy, typically large currency and interest rate shifts. These funds make extensive use of leverage and derivatives.

Emerging Markets [3%]: Funds that focus on emerging and less mature markets. These funds tend to be long only because in many emerging markets short selling is not permitted and futures and options are not available.

The database used in this study covers the period June 1994–May 2001 and was obtained from Tremont TASS, which is one of the best known and largest hedge fund databases available. Our database includes the Asian, Russian and LTCM crises as well as the end of the IT bubble and the first part of the bear market that followed. As of May 2001, the database contains monthly net of fee returns on a total of 2183 hedge funds and FoHF. Reflecting the tremendous growth of the industry as well as a notoriously high attrition rate, only 264 of these hedge funds had seven or more years of data available.

As shown in Amin and Kat [2003a], concentrating on surviving funds only will not only overestimate the mean return on individual hedge funds by around 2% but will also introduce significant biases in estimates of standard deviation, skewness and kurtosis. To avoid this problem we decide not to work with the raw return series of the 264 survivor funds but instead to create 348 seven-year monthly return series by, starting off with the 348 funds that were alive in June 1994, replacing every fund that closed down during the sample period by a fund randomly selected from the set of funds alive at the time of closure following the same type of strategy and of similar size and age. We do not include FoHF or dedicated short bias funds in our sample.

This replacement procedure implicitly assumes that in case of fund closure investors are able to roll from one fund into the other at the reported end-of-month net asset value and at zero additional costs. This somewhat underestimates the true costs of fund closure to investors for two reasons. First, when a fund closes shop its investors have to look for a replacement investment. This search takes time and is not without costs. Second, investors may get out of the old and into the new fund at values that are less favorable than the end-of-month net asset values contained in the database. Unfortunately, it is impossible to correct for this without additional information.

As hedge funds frequently invest in, to various degrees and combinations, illiquid exchange-traded and difficult-to-price over-the-counter securities, hedge fund administrators can have great difficulty in marking a portfolio to market at the end of the month to arrive at the fund's net asset value. Having difficulty obtaining an accurate value for illiquid assets, most will rely on 'old' prices or observed transaction prices for similar but more liquid assets. Such partial adjustment or 'smoothing' produces systematic valuation errors which tend not to be diversified away, resulting in serial correlation in monthly returns and underestimation of their true standard deviations. In this paper we follow the approach of Brooks and Kat [2001], outlined in the appendix, to unsmooth hedge fund returns and thereby reconcile stale price problems. Table 1 provides a statistical summary of reported and unsmoothed individual hedge fund returns. Looking at the 1-month autocorrelations, the smoothing problem is especially acute among convertible arbitrage and distressed securities funds. This is plausible as the securities held by these funds tend to be highly illiquid. As well, the unsmoothing produces standard deviations that are substantially higher than those calculated from reported returns, especially in convertible arbitrage and distressed securities where we observe a rise of around 30%. In what follows we concentrate on the unsmoothed returns.

Table 1 offers some other insights as well. Funds in different strategy groups tend to generate quite different returns, which confirms that the (self-)classification used has significant discriminatory power. From the table it is also clear that the risk profile of the average hedge fund cannot be accurately described by standard deviation alone. The table reports that the majority of funds in each strategy group reject the null hypothesis of a normal return distribution under a Jarque-Bera test at the 5% significance level. All strategy groups exhibit non-zero skewness and excess kurtosis, with global macro being the only strategy producing positive skewness.

Most existing research on optimal hedge fund allocation, including Lamm [2003], Morton et al. [2006], Amenc and Martellini [2002] and Cvitanić et al. [2003], uses well-known hedge fund indices (obtained from, for example, HFR or CSFB-TASS) to represent the different hedge fund strategy classes. Given the different number of funds in each index, however, different indices will achieve different levels of diversification. As shown in Amin and Kat [2002] and Davies, Kat and Lu [2003], hedge fund portfolio return properties vary substantially with the number of hedge funds included in the portfolio. For instance, an index constructed from only ten funds will typically have significantly higher variance than a similar index constructed from 100 funds. An index composed of more funds is therefore likely to be allocated more capital. This higher allocation, however, results because this *index* has less specific risk than other indices based on a smaller number of funds, rather than because this *strategy* has lower risk.

To compare "like for like," we construct representative portfolios containing the same number of funds for each hedge fund strategy. Specifically, we consider representative portfolios of 5 and 15 funds to capture the feasible investment possibilities of small and large sized FoHF. In practice, FoHF have to deal with minimum investment requirements, typically ranging from $100,000 to $500,000 per fund. For smaller FoHF this forms a significant barrier to diversification. In contrast, large FoHF typically spread their investments over a relatively large number of managers to prevent the fund from becoming the dominant investor in any one particular fund.[7]

We construct the representative portfolios for a given strategy as follows. First, we randomly sample 5000 portfolios of a given size (5 or 15 funds). We calculate each portfolio's mean, standard deviation, skewness and kurtosis and take the average of each moment over the 5000 portfolios. The representative portfolio is then selected from the 5000 random

[7] The optimal number of funds (within a strategy group) is an interesting area which will be dealt with in a subsequent paper. In part, the number of funds reflects a trade-off between possible diversification benefits and the cost of finding and monitoring high quality funds.

portfolios in order to minimize the sum of the ranked differences across each of the four average moments. Table 2 provides the return characteristics of the representative portfolios thus obtained. In unreported results, we also considered representative portfolios of 10 and 20 funds. Our results show that as the number of funds in portfolio increases, standard deviations fall substantially. This indicates relatively low correlation between funds within the same strategy group, i.e. a high level of fund specific risk. Also, when the number of funds increases, portfolio return distributions become more skewed, indicating a high degree of co-skewness between funds within the same strategy group. Diversification is no longer a free lunch: Investors pay for a lower standard deviation by accepting a lower level of skewness. The only exception is global macro, where lower standard deviations go hand in hand with higher levels of skewness.

In practice, managers of FoHF do not select hedge funds by random sampling. That said, the fact that many spend a lot of time and effort to select the funds they invest in does not necessarily mean that in many cases a randomly sampled portfolio is not a good proxy for the portfolio that is ultimately selected. There is no evidence that some FoHF are able to consistently select future outperformers, nor is there any evidence of specific patterns or anomalies in hedge fund returns. When properly corrected for all possible biases there is no significant persistence in hedge fund returns, nor is there any significant difference in performance between older and younger funds, large and small funds, etc. In addition, older funds may be more or less closed to new investment, implying that expanding fund of funds are often forced to invest in funds with little or no track record. The fund prospectus and manager interviews may provide some information, but in most cases this information will be sketchy at best and may add more noise than actual value.

Finally, our analysis uses the sample average of the 90-day US T-bill rate, $r = 0.423317\%$ on a monthly basis, as the risk-free rate. While for the most part we focus on the portfolio selected by a FoHF that neither borrows nor lends, the risk-free rate is necessary to determine the optimal portfolio since it reflects the leverage possibilities available to a FoHF investor.

3 Empirical Results

We now use PGP optimization to obtain optimal portfolios for ten different sets of investor preferences, for both FoHF portfolios and portfolios of stocks, bonds, and hedge funds. These preference sets are chosen to illustrate the extent to which investors must trade off different moments, to determine which hedge fund strategies are crucial in determining overall portfolio performance, and to see how allocations change if we impose capital constraints on each hedge fund strategy.

3.1 Trade-Off Between Multiple Objectives

The more importance investors attach to a certain moment, i.e. the greater the preference parameter for this moment, the more favorable the value of this moment statistic will tend to be in the optimal portfolio. Investor preferences (α, β, γ) determine the relative importance of the difference between the values for expected return, skewness, and kurtosis obtained in mean-variance-skewness-kurtosis space and their corresponding optimal values obtained in mean-variance (Z_1^*), skewness-variance (Z_3^*), and kurtosis-variance space (Z_4^*). Figure 1 shows that the difference, $d_1 = Z_1^* - E\left[\mathbf{X}^\top \tilde{\mathbf{R}}\right] - x_{n+1}r$, decreases monotonically as investors' preference for expected return (α) increases, holding $\beta = 1$ and $\gamma = 0.5$ fixed. It also provides the analogous results for skewness (holding $\alpha = 1$ and $\gamma = 0.5$ fixed) and kurtosis (holding $\alpha = 1$ and $\beta = 1$ fixed). Based on these results, we choose realistic values for the preference parameters: $\alpha, \beta \in \{0 \text{ (none)}, 1 \text{ (low)}, 2 \text{ (medium)}, 3 \text{ (high)}\}$ and $\gamma \in \{0 \text{ (none)}, 0.25 \text{ (low)}, 0.5 \text{ (medium)}, 0.75 \text{ (high)}\}$, to capture investors with no, low, medium, and high preference, respectively, for the applicable return moment.

Table 3 provides the return characteristics of PGP optimal portfolios for small and large investors for different sets of investor preferences over expected return (α), skewness (β) and kurtosis (γ). Portfolio A with $(\alpha, \beta, \gamma) = (1, 0, 0)$ corresponds with the mean-variance efficient portfolio. Expected return is relatively high and standard deviation low, which in mean-variance terms makes for a highly attractive portfolio. Looking beyond mean and variance, however, we see that the skewness and kurtosis properties of this portfolio are extremely unattractive. This confirms the point raised by Amin and Kat [2003b] that mean-variance optimizers may be nothing more than skewness minimizers.

Portfolios B–J show that variations in investor preferences will change the risk-return characteristics of the optimal portfolio to quite an extent. These results reinforce the trade-offs illustrated in figure 1 and show that as one moment statistic improves, at least one of the other three moment statistics will tend to deteriorate. Compare, for example, portfolio E, $(\alpha, \beta, \gamma) = (1, 3, 0.25)$, with portfolio H, $(\alpha, \beta, \gamma) = (2, 3, 0.25)$. These two portfolios have the same level of preference over skewness and kurtosis. Despite this, the higher preference over expected return in portfolio H leads to a higher expected return at the cost of a higher standard deviation. The same phenomenon can be observed by comparing portfolio G, $(\alpha, \beta, \gamma) = (2, 1, 0.75)$, with portfolio H. Higher preference for skewness and lower preference for kurtosis causes the high kurtosis and high standard deviation of portfolio H to be traded in for a higher expected return and substantially higher skewness.

The above observation that hedge fund moment statistics tend to trade off against each other is quite an interesting one. Despite the fact that hedge funds often follow highly active, complex strategies, hedge fund returns seem to exhibit the same type of trade-offs typically observed in the underlying securities markets, where prices are (thought to be) explicitly set to generate this type of phenomenon. Hedge funds therefore appear unable to dodge the rules of the game and seem to pick up a lot more from the markets that they trade than their well-cultivated market neutral image may suggest.

Figure 2 shows the feasible set of portfolios and the resulting mean-variance efficient frontier as well as the mean-variance coordinates of some specific optimal portfolios, including the mean-variance-skewness efficient portfolio (portfolio B with $(\alpha, \beta, \gamma) = (1, 1, 0)$), and the mean-variance-kurtosis efficient portfolio (with $(\alpha, \beta, \gamma) = (1, 0, 1)$). Doing so demonstrates a key point of our analysis: if investor preferences over skewness and kurtosis are incorporated into the portfolio decision, then in mean-variance space the optimal portfolio may well lie *below* the mean-variance efficient frontier. The reason is, of course, that expected return, skewness, and kurtosis are conflicting objectives. Portfolios with relatively high skewness and low kurtosis will tend to come with a relatively low expected rate of return and vice versa.

Even though investor preferences over variance (or standard deviation) are not explicitly specified in our objective function, variance still plays a key role in the tradeoff interaction. In the first stage of our PGP optimization, the optimal values of Z_1^*, Z_3^*, and Z_4^* are each obtained by seeking the best tradeoff between variance and return, variance and skewness, and variance and kurtosis, respectively. In the second stage of the PGP optimization, we obtain the optimal portfolio that has the best possible expected return, skewness and kurtosis with the relative tradeoffs between them determined by how close their values are to Z_1^*, Z_3^* and Z_4^* with the difference "penalty" determined by investor preferences. Standard deviation is therefore essentially the tradeoff counterpart to every moment in the optimization process.

3.2 Optimal Allocation Across Hedge Fund Strategies

Table 3 also reports the optimal allocation weights across the different hedge fund strategies for different sets of investor preferences over expected return, skewness, and kurtosis. When the analysis is limited to mean-variance space (portfolio A), merger arbitrage is allocated a dominant 80%. This, however, fully reflects merger arbitrage's comparatively low volatility and high return during our data period. The attractive mean-variance characteristics of merger arbitrage come at the cost of unfavorable skewness and kurtosis properties. When preference for skewness and kurtosis is introduced, the allocations change dramatically. The allocation to merger arbitrage drops to a much lower level, while global macro and equity market neutral take over as the dominant strategies, irrespective of investor size or preferences. Convertible arbitrage and long/short equity tend to receive relatively small allocations here and there. No money is allocated to distressed securities and emerging markets.

The allocations in table 3 are not at all in line with strategies' means and variances as reported in table 2. Purely based on mean and variance, one would expect a much higher allocation to merger arbitrage, convertible arbitrage, long/short equity and distressed securities. Part of the explanation lies in the skewness and kurtosis values reported in table 2. Global macro combines positive skewness with low kurtosis. Merger arbitrage, convertible arbitrage, and especially distressed securities, however, exhibit exactly the opposite characteristics.

It is well known that in diversified portfolios the marginal return characteristics of the assets involved only play a relatively minor role in determining the return characteristics of the portfolio. To explain the above allocations we therefore must look not only at the various strategies' marginal return properties (as given by table 2) but also, and especially, at the way they are related to each other. Doing so may explain why for example in the PGP optimal portfolios equity market neutral, which offers a low expected return and significant negative skewness, receives a higher allocation than long/short equity, which offers a high expected return and less skewness.

As shown in Davies, Kat and Lu [2003], long/short equity tends to exhibit negative co-skewness and high co-kurtosis with other strategies. This means that in a portfolio context the (negative) impact of long/short equity on portfolio skewness and kurtosis will be stronger than evident from its marginal statistics. Equity market neutral on the other hand tends to exhibit low co-variance and low co-kurtosis with other strategies. This makes this strategy attractive as a volatility and kurtosis reducer, which is reflected in the allocations, especially when preference for kurtosis is high as in portfolio F and portfolio G. Global macro tends to exhibit positive co-skewness with other strategies and thereby acts as portfolio skewness enhancer, which explains why this particular strategy picks up by far the highest allocations, particularly when there is a strong preference for skewness such as in portfolio E and portfolio H. Contrary to global macro, distressed securities displays strong negative co-skewness with other strategies, which explains the complete lack of allocations to the latter strategy.

From an economic perspective, none of the above comes as a complete surprise. Although many hedge funds do not invest directly in equities, a significant drop in stock prices is often accompanied by a widening of credit spreads, a significant drop in market liquidity, higher volatility, etc. Since hedge fund returns are highly sensitive to these factors, most of them will perform poorly when there is a fall in stock prices, which will technically show up as negative co-skewness.[8] The recent bear market provides a good example. Over the 3 years that stock prices dropped, overall hedge fund performance (as measured by the main indices) was virtually flat. The main exceptions to the above are equity market neutral and global macro funds. For equity market neutral funds maintaining market neutrality is one of their prime goals, which makes them less sensitive to market moves than other funds. Global macro funds tend to take views on macro economic events and are generally thought to perform best when markets drop and/or become more volatile, which is confirmed by their positive co-skewness properties. Convertible arbitrage funds, which are long convertibles, will suffer when stock markets come down. On the other hand, they will benefit from the simultaneous increase in volatility. Overall, this provides convertible arbitrage with relatively moderate risk characteristics, which in turn explains the allocations to this particular strategy.[9]

At this stage, it is important to emphasize that the PGP technique is general enough to accommodate an investor that wishes to replace historic return distributions with her own beliefs. For instance, an investor could first use the method proposed by Black and Litterman [1990, 1992] to combine market equilibrium implied returns with her own subjective views and then use the resulting mixed estimate of expected returns to re-center the distribution of hedge fund returns, while maintaining its overall shape (in terms of variance, skewness, and kurtosis). Clearly, this approach would lead to different allocations than those reported here. For example, it would likely increase expected returns for emerging market funds and thereby cause this strategy to receive a higher allocation under some preference parameters.

We next consider the sensitivity of the allocations to constraints on standard deviation,

[8]Note that this also implies a strong negative co-skewness between hedge funds and the stock market. We will return to this point in section 3.4.

[9]With more and more convertible arbitrage funds competing for the same trades, some funds may decide to no longer hedge their credit risk exposure to compensate for the loss of margin. Those funds can be expected to exhibit a more aggressive risk profile, especially lower co-skewness with other funds and equity.

skewness, and kurtosis.[10] Part A of Tables 4–5 illustrates optimal allocations for portfolios that have the same preferences over objectives as those in Table 3, but under the constraint that the portfolio standard deviation is 10% less than its unconstrained value. Part B illustrates the case in which the value of each portfolio's skewness is constrained to be 10% higher than its counterpart in Table 3 and part C illustrates the case in which each portfolio's kurtosis is constrained to be 10% lower than its counterpart in Table 3. In the tables, equity market neutral's role in reducing portfolio standard deviation and kurtosis and global macro's role as a skewness enhancer is evident. Allocations to global macro increase when portfolio skewness is constrained, while allocations to equity market neutral increase when standard deviation or kurtosis are constrained.

We have shown that global macro and equity market neutral funds have important roles to play in FoHF portfolios. More generally, our analysis suggests that it may be optimal for FoHF to concentrate on just a few specific strategies rather than diversify across a large variety of them. In practice, strategy-focused FoHF, however, are a lot less common than well-diversified FoHF.

3.3 Constraints on Capital Allocations

Our framework easily accommodates restrictions on the allocations to a hedge fund strategy. To illustrate, we consider the case where the allocation to each hedge fund strategy is constrained to be no more than 30% of total capital ($x_i \leq 0.30 \; \forall i$). Table 6 displays the resulting optimal allocations. When allocations are constrained, the degree of variation in return parameters achievable is quite a lot less than in the unconstrained case. This underlines that significant improvements in portfolio skewness and kurtosis can only be achieved by restricting the number of hedge fund strategies. In comparison with Table 3, we notice that the previously dominant capital weights on equity market neutral funds and global

[10] In practice, these constraints could reflect the real or perceived need for fund managers, particularly new ones, to match closely the risk profile of the fund's peer group. They could also reflect constraints explicitly imposed by the fund's investors.

macro funds are now forced down to 30%, with merger arbitrage and convertible arbitrage picking up the difference. As in the case without constraints, the model continues to avoid distressed securities, long/short equity and emerging markets.

3.4 Portfolios of Stocks, Bonds and Hedge Funds

Until now, we have studied FoHF portfolios in isolation, i.e. implicitly assuming investors will not invest in anything else than the FoHF. In practice, however, investors will mix FoHF in with their existing portfolio. The means that preferably the optimal fund of funds portfolio should be derived from a wider framework, including stocks and bonds, to take account of the relation between hedge funds, stocks and bonds. This is what we do in this section.

Table 7 reports the moment statistics of optimal portfolios constructed with stocks, bonds, and hedge fund strategies for small and large investors. Although the addition of stocks and bonds results in optimal portfolios with less kurtosis and higher skewness than the corresponding optimal portfolios of hedge funds only, we observe similar behavior. Again, the mean-variance efficient portfolio has relatively unattractive skewness and kurtosis properties, which improve when explicit preference for higher moments is introduced. The improvement comes at a cost, however, as moment statistics tend to trade off against each other. An improvement in one statistic can only be obtained by accepting deterioration in one or more others.

Table 7 reveals that the bond index is the primary recipient of capital, receiving at least 40% weight in most instances. In stark contrast, the stock index is sold short, irrespective of the investor preference parameters. This result is consistent with the observation of Amin and Kat [2003b] and Davies, Kat and Lu [2003] that hedge funds mix far better with bonds than with stocks. Whereas the co-skewness between stocks and most hedge fund strategies is negative, the co-skewness between bonds and hedge funds is generally higher and the co-kurtosis lower. A long bonds position and a short position in stocks combined with positive holdings in hedge funds will increase portfolio skewness and reduce kurtosis. As a result, optimal portfolios have to rely less on equity market neutral and global macro to perform these tasks, which allows them to diversify into strategies such as long/short equity and merger arbitrage. Even in this new context, however, no allocation is given to distressed securities and emerging markets strategies.

Table 8 shows the asset allocation across stocks, bonds and hedge funds under the constraint that capital weights are between zero and 30% for any hedge fund strategy and that the capital weights are within ±30% for both the stock index and the bond index. In general, these constraints seem to have quite an impact on the optimal portfolio moment statistics, reflecting a substantial change in the underlying allocations. With the bond allocation restricted to 30%, the optimal portfolios turn increasingly towards equity market neutral and global equity to control variance, skewness and kurtosis. Investment in equity market neutral and global macro funds seems to make a similar contribution to the overall portfolio return distribution as an investment in the bond index.

4 Conclusion

This paper has incorporated investor preferences for higher moments into a PGP optimization function. This allows us to solve for multiple competing (and often conflicting) hedge fund allocation objectives within a 4-moment framework. Our empirical analysis has yielded a number of conclusions, the most important being that:

- Hedge fund return moment statistics tend to trade off against each other in much the same way as in the underlying securities markets. Despite following often complex strategies, hedge funds therefore appear unable to dodge the rules of the game. This is in line with the results of Amin and Kat [2003c] who conclude that when taking the entire return distribution into account there is nothing superior about hedge fund returns.

- Introducing preferences for skewness and kurtosis in the portfolio decision-making process may yield portfolios far different from the mean-variance optimal portfolio, with much less attractive mean-variance characteristics. This again emphasizes the various trade-offs involved.

- Equity market neutral and global macro funds have important roles to play in optimal hedge fund portfolios thanks to their attractive co-variance, co-skewness and co-kurtosis properties. Equity market neutral funds act as volatility and kurtosis reducers, while global macro funds act as skewness enhancers.
- Especially in terms of skewness, hedge funds and stocks do seem not combine very well. This suggests that investors may be better off using hedge funds to replace stocks instead of bonds, as appears to be current practice.

A Procedure to "unsmooth" data

The observed (or smoothed) value V_t^* of a hedge fund at time t can be expressed as a weighted average of the underlying (true) value at time t, V_t, and the smoothed value at time $t-1$, V_{t-1}^*: $V_t^* = \alpha V_t + (1-\alpha)V_{t-1}^*$. Let B be the backshift operator defined by $B^L x_t = x_{t-L}$. Define the following lag function, $L_t(\alpha)$, which is a polynomial B, with different coefficients for each of the $t = 1, \ldots, 12$ appraisal cohorts:

$$L_t(\alpha) = \frac{t}{12} + \sum_{L=1}^{\infty} \left[(1-\alpha)^{L-1} \left(\frac{12-t}{12} \right) + (1-\alpha)^L \left(\frac{t}{12} \right) \right] B^L.$$

Let r_t and r_t^* denote the true underlying (unobservable) return and the observed return at time t respectively. The monthly smoothed return is given by $r_t^* = \alpha L_m(\alpha) r_t$. We then can derive:

$$r_t^* = \alpha r_t + (1-\alpha) r_{t-1}^* = \alpha r_t + \alpha(1-\alpha) r_{t-1} + \alpha(1-\alpha)^2 r_{t-2} \cdots. \tag{12}$$

Here we implicitly assume that hedge fund managers use a single exponential smoothing approach. This yields an unsmoothed series with zero first order autocorrelation: $r_t = \alpha^{-1}(r_t^* - (1-\alpha) r_{t-1}^*)$. Since the stock market indices have around zero autocorrelation coefficients, it seems plausible in the context of the results above to set $1 - \alpha$ equal to the first order autocorrelation coefficient. The newly constructed return series, r_t, has the same mean as r_t^*, and zero first order autocorrelation (aside from rounding errors), but with higher standard deviation.

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Table 1: **Statistical summary of reported and "unsmoothed" hedge fund returns and stock/bond returns.** Reported values are calculated from monthly net-of-all-fee returns and averaged across funds. "Unsmoothed" returns are reported returns adjusted for possible stale price effects using the technique described in the appendix. Kurtosis measures excess kurtosis. First-order to fourth-order autocorrelation is given by AC(1)–AC(4). The final column reports the percentage of funds within each strategy that reject the null hypothesis of a normal return distribution under a Jarque-Bera (J-B) test with a 5% significance level.

Based on reported returns

	N	Mean	Std Dev	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)	AC(4)	J-B (%)
Convertible arbitrage	24	0.96	3.01	−1.14	5.93	0.30	0.15	0.09	0.02	91.7
Distressed securities	29	0.89	2.37	−0.78	6.36	0.25	0.08	−0.04	0.02	86.2
Equity market neutral	12	0.54	2.70	−0.41	2.82	0.20	0.03	0.05	0.05	50.0
Global macro	46	0.77	5.23	1.06	7.63	0.11	0.01	−0.00	−0.03	84.8
Long/short equity	172	1.34	5.83	0.00	3.35	0.09	−0.00	0.01	−0.03	69.2
Merger arbitrage	18	1.17	1.75	−0.50	4.96	0.10	−0.00	0.00	−0.03	77.8
Emerging markets	47	0.22	7.85	−0.86	5.79	0.10	−0.01	−0.00	−0.02	68.1
S&P500		1.36	4.39	−0.83	1.11	−0.11	−0.05	0.03	−0.06	
Bond index		0.59	0.84	0.24	1.39	0.22	0.12	0.06	0.02	

Based on "unsmoothed" returns

	N	Mean	Std Dev	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)	AC(4)	J-B (%)
Convertible arbitrage	24	0.96	3.99	−0.91	5.46	0.00	−0.03	−0.01	−0.02	79.2
Distressed securities	29	0.91	3.06	−0.67	6.60	0.01	−0.03	−0.01	−0.02	86.2
Equity market neutral	12	0.55	3.06	−0.39	2.94	0.01	−0.03	−0.01	−0.02	50.0
Global macro	46	0.76	5.35	1.03	7.16	0.01	−0.02	−0.01	−0.03	82.6
Long/short equity	172	1.37	6.35	0.01	3.19	0.00	−0.03	−0.00	−0.03	68.6
Merger arbitrage	18	1.17	2.06	−0.46	4.65	0.00	−0.03	−0.01	−0.02	77.8
Emerging markets	47	0.23	9.63	−0.91	5.91	0.01	−0.03	−0.01	−0.02	66.0

Table 2: **Statistical summary of returns for representative portfolios.** Representative portfolios are obtained by first randomly sampling 5000 portfolios of a given size (5 or 15 funds). The mean, standard deviation, skewness and kurtosis of each randomly sampled portfolio's return series is calculated. Then, the average of each moment over the 5000 portfolios is taken. The representative portfolio is then selected from the 5000 random portfolios in order to minimize the sum of the ranked differences across each of the four average moments. Reported values are calculated from monthly net-of-all-fee returns and averaged across funds. Kurtosis measures excess kurtosis.

	Mean	**Std Dev**	**Skewness**	**Kurtosis**
Small Investors (each strategy portfolio has 5 funds)				
Convertible arbitrage	0.96	2.84	−0.83	5.11
Distressed securities	0.89	2.23	−1.89	11.00
Equity market neutral	0.50	1.75	−0.40	1.53
Global macro	0.75	3.37	0.64	2.41
Long/short equity	1.38	4.03	−0.21	2.10
Merger arbitrage	1.16	1.54	−1.08	6.62
Emerging markets	0.25	7.54	−1.15	5.80
Large Investors (each strategy portfolio has 15 funds)				
Convertible arbitrage	0.95	2.35	−1.00	4.65
Distressed securities	0.90	2.03	−2.55	14.31
Equity market neutral	0.52	1.31	−0.65	1.26
Global macro	0.75	2.79	0.76	1.64
Long/short equity	1.37	3.55	−0.23	1.83
Merger arbitrage	1.17	1.37	−1.71	8.98
Emerging markets	0.22	7.19	−1.27	5.93

Table 3: **Moment statistics and asset allocation across strategy classes for optimal FoHF portfolios.** Reported values are calculated from monthly net-of-all-fee returns. Investors assign capital to representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage, and emerging market hedge funds. Optimal allocations are based on investors' preferences over expected return (α), skewness (β) and kurtosis (γ) using the PGP technique.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
Small investors (representative strategy portfolios have 5 funds)										
Moments:										
Expected return	1.07	0.85	1.38	0.83	0.79	0.70	0.66	0.84	0.79	0.77
Standard deviation	1.30	2.78	4.03	1.42	2.40	1.59	1.46	2.84	1.70	1.38
Skewness	−1.25	0.66	−0.21	0.25	0.63	0.27	0.23	0.66	0.51	0.15
Kurtosis	6.07	1.96	2.10	1.01	1.76	−0.26	−0.27	1.72	1.35	0.05
Allocation:										
Convertible arbitrage	0.05	0.04	0.00	0.10	0.13	0.11	0.13	0.03	0.09	0.11
Distressed securities	0.00	0.00	0.00	0.00	0.04	0.00	0.10	0.00	0.00	0.00
Equity market neutral	0.10	0.01	0.00	0.27	0.12	0.52	0.51	0.04	0.28	0.41
Global macro	0.05	0.78	0.00	0.32	0.66	0.28	0.25	0.78	0.43	0.25
Long/short equity	0.00	0.10	1.00	0.00	0.05	0.09	0.00	0.15	0.01	0.04
Merger arbitrage	0.80	0.07	0.00	0.30	0.00	0.00	0.00	0.00	0.19	0.19
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Large investors (representative strategy portfolios have 15 funds)										
Moments:										
Expected return	0.97	0.75	0.76	0.85	0.73	0.66	0.77	0.75	0.78	0.79
Standard deviation	0.92	1.51	1.40	1.13	1.69	1.35	0.92	1.50	1.26	0.93
Skewness	−1.29	0.83	0.80	0.31	0.85	0.42	0.19	0.83	0.72	0.21
Kurtosis	6.19	1.42	1.29	1.13	1.42	0.19	0.29	1.40	1.20	0.45
Allocation:										
Convertible arbitrage	0.02	0.01	0.04	0.09	0.00	0.16	0.09	0.02	0.03	0.08
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.30	0.32	0.34	0.27	0.32	0.45	0.47	0.32	0.34	0.44
Global macro	0.00	0.49	0.44	0.30	0.56	0.35	0.19	0.49	0.38	0.20
Long/short equity	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	0.68	0.18	0.18	0.34	0.09	0.00	0.25	0.17	0.25	0.29
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00

Table 4: **Asset allocation for optimal hedge fund portfolios with constrained portfolio standard deviation, skewness or kurtosis for small investors.** Panel A illustrates optimal allocations for portfolios that have the same preferences over objectives as those in Table 4, but under the constraint that the portfolio standard deviation is 10% less than its unconstrained value. Panel B illustrates the case in which the value of each portfolio's skewness is constrained to be 10% higher than its counterpart in Table 4 and panel C illustrates the case in which each portfolio's kurtosis is constrained to be 10% lower than its counterpart in Table 4. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β), and excess kurtosis (γ) using the PGP technique, selected from representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Each representative strategy portfolio has 5 funds. Bold numbers indicate that the strategy capital loading has increased relative to the optimal unconstrained portfolio.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
A: Standard deviation constrained (10% improvement)										
Convertible arbitrage	**0.06**	**0.05**	**0.07**	0.09	0.00	**0.12**	0.11	**0.05**	**0.13**	0.06
Distressed securities	0.00	0.00	0.00	0.00	0.00	**0.11**	0.00	0.00	0.00	**0.07**
Equity market neutral	**0.21**	**0.07**	**0.33**	**0.35**	**0.68**	**0.53**	**0.47**	**0.13**	**0.30**	**0.63**
Global macro	0.05	0.70	**0.47**	0.25	0.07	0.24	0.22	0.69	0.35	0.05
Long/short equity	0.00	0.07	0.13	0.00	0.01	0.00	0.00	0.13	0.01	0.00
Merger arbitrage	0.68	**0.10**	0.00	0.31	0.24	0.00	**0.20**	0.00	**0.21**	0.18
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B: Skewness constrained (10% improvement)										
Convertible arbitrage	**0.09**	0.01	**0.07**	0.10	0.01	0.11	0.12	0.01	0.08	0.11
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	**0.13**	0.00	**0.33**	0.26	0.00	0.52	0.50	0.00	0.16	0.40
Global macro	**0.13**	**0.89**	0.48	**0.34**	**0.89**	**0.29**	**0.28**	**0.90**	**0.53**	**0.26**
Long/short equity	0.00	**0.12**	0.13	0.00	**0.11**	0.09	**0.11**	0.12	**0.06**	0.02
Merger arbitrage	0.65	0.00	0.00	0.30	0.00	0.00	0.00	0.00	0.17	**0.21**
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C: Kurtosis constrained (10% improvement)										
Convertible arbitrage	**0.08**	0.04	**0.07**	**0.12**	0.02	0.11	**0.14**	**0.04**	0.08	0.10
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00
Equity market neutral	**0.13**	**0.03**	**0.33**	**0.29**	0.12	**0.53**	**0.54**	**0.12**	0.24	**0.42**
Global macro	0.04	0.78	**0.48**	**0.38**	**0.71**	0.27	0.25	0.69	**0.48**	0.25
Long/short equity	0.00	**0.14**	0.13	**0.16**	**0.15**	0.09	**0.03**	0.15	**0.09**	0.02
Merger arbitrage	0.75	0.01	0.00	0.04	0.00	0.00	0.00	0.00	0.11	**0.21**
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 5: **Asset allocation for optimal hedge fund portfolios with constrained portfolio standard deviation, skewness or kurtosis for large investors.** Panel A illustrates optimal allocations for portfolios that have the same preferences over objectives as those in Table 4, but under the constraint that the portfolio standard deviation is 10% less than its unconstrained value. Panel B illustrates the case in which the value of each portfolio's skewness is constrained to be 10% higher than its counterpart in Table 4 and panel C illustrates the case in which each portfolio's kurtosis is constrained to be 10% lower than its counterpart in Table 4. Each optimal portfolio is constructed under investors' preferences over expected return (α), skewness (β), and excess kurtosis (γ) using the PGP technique, selected from representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage and emerging market funds. Each representative strategy portfolio has 15 funds. Bold numbers indicate that the strategy capital loading has increased relative to the optimal unconstrained portfolio.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
A: Standard deviation constrained (10% improvement)										
Convertible arbitrage	**0.03**	**0.04**	0.04	0.00	**0.02**	0.14	0.07	0.02	0.03	0.03
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.01**
Equity market neutral	**0.39**	**0.49**	**0.38**	**0.46**	**0.34**	**0.47**	**0.57**	**0.36**	**0.38**	**0.62**
Global macro	**0.01**	0.13	0.38	0.23	0.50	0.30	0.00	0.42	0.32	0.00
Long/short equity	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**0.02**
Merger arbitrage	0.57	**0.34**	**0.20**	0.32	**0.15**	**0.06**	**0.36**	**0.19**	**0.26**	**0.32**
Emerging markets	0.00	0.00	0.00	0.00	0.00	**0.03**	0.00	0.00	0.00	0.00
B: Skewness constrained (10% improvement)										
Convertible arbitrage	**0.03**	0.01	0.00	0.05	0.00	**0.18**	0.09	0.00	0.02	0.07
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	**0.33**	0.32	0.30	**0.38**	0.30	0.26	0.47	0.31	0.33	0.43
Global macro	0.00	0.49	**0.62**	0.26	**0.61**	**0.48**	0.19	**0.61**	**0.44**	0.20
Long/short equity	0.00	0.00	**0.03**	0.00	0.02	**0.05**	0.00	**0.02**	0.00	0.00
Merger arbitrage	0.65	0.18	0.06	0.31	0.08	0.00	0.25	0.06	0.21	0.29
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
C: Kurtosis constrained (10% improvement)										
Convertible arbitrage	**0.03**	**0.16**	0.03	**0.13**	0.00	0.10	0.07	0.02	**0.05**	0.08
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	**0.31**	**0.46**	0.34	**0.83**	0.32	**0.50**	**0.57**	**0.34**	**0.36**	0.44
Global macro	0.00	0.34	**0.47**	0.03	**0.58**	0.17	0.00	0.48	0.36	0.19
Long/short equity	0.00	0.00	**0.05**	0.00	**0.07**	0.00	0.00	**0.03**	0.00	0.00
Merger arbitrage	0.66	0.00	0.12	0.01	0.03	0.22	**0.36**	0.14	0.24	0.29
Emerging markets	0.00	**0.04**	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6: **Moment statistics and asset allocation across strategy classes for optimal fund of hedge fund portfolios with global constraints on capital investment.** Reported values are calculated from monthly net-of-all-fee returns. Investors assign capital to representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage, emerging market hedge funds. Optimal allocations are based on investors' preferences over expected return (α), skewness (β) and kurtosis (γ) using the PGP technique. The capital weight for each hedge fund strategy is constrained to be between zero and 30%.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
Small investors (representative strategy portfolios have 5 funds)										
Moments:										
Expected return	0.90	0.82	0.81	0.82	0.81	0.84	0.76	0.81	0.82	0.82
Standard deviation	1.28	1.39	1.40	1.38	1.40	1.76	1.61	1.39	1.39	1.42
Skewness	−1.26	0.22	0.22	0.21	0.22	0.08	0.04	0.22	0.22	0.19
Kurtosis	3.47	0.74	0.64	0.80	0.67	0.12	0.55	0.69	0.73	0.59
Allocation:										
Convertible arbitrage	0.19	0.12	0.15	0.10	0.14	0.19	0.30	0.13	0.12	0.14
Distressed securities	0.15	0.00	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00
Equity market neutral	0.24	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Global macro	0.07	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Long/short equity	0.04	0.00	0.00	0.00	0.00	0.19	0.01	0.00	0.00	0.02
Merger arbitrage	0.30	0.28	0.25	0.30	0.26	0.02	0.02	0.27	0.28	0.24
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Large investors (representative strategy portfolios have 15 funds)										
Moments:										
Expected return	0.88	0.83	0.83	0.83	0.83	0.81	0.82	0.83	0.83	0.83
Standard deviation	1.00	1.12	1.12	1.12	1.12	1.15	1.14	1.12	1.12	1.12
Skewness	−1.32	0.39	0.39	0.39	0.39	0.32	0.34	0.39	0.39	0.39
Kurtosis	3.63	0.91	0.86	0.91	0.91	0.59	0.63	0.91	0.91	0.85
Allocation:										
Convertible arbitrage	0.18	0.10	0.11	0.10	0.10	0.17	0.16	0.10	0.10	0.11
Distressed securities	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Global macro	0.06	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Long/short equity	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	0.30	0.30	0.29	0.30	0.30	0.23	0.24	0.30	0.30	0.29
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7: **Moment statistics and optimal asset allocation across stocks, bonds, and hedge fund strategies.** Reported values are calculated from monthly net-of-all-fee returns. Investors assign capital to representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage, emerging market hedge funds, the S&P 500 index, and the Salomon Brothers 7-Year Government Bond US Index. Optimal allocations are based on investors' preferences over expected return (α), skewness (β) and kurtosis (γ) using the PGP technique.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
Small investors (representative strategy portfolios have 5 funds)										
Moments:										
Expected return	0.83	0.87	0.72	0.76	0.72	0.60	0.74	0.56	0.75	0.75
Standard deviation	0.71	3.20	0.87	0.63	0.88	0.75	0.61	0.77	0.73	0.62
Skewness	−0.14	0.88	0.70	0.13	0.71	0.16	0.11	0.48	0.48	0.10
Kurtosis	0.94	3.48	1.17	−0.21	1.22	−0.81	−0.40	0.82	0.61	−0.33
Allocation:										
Convertible arbitrage	0.06	0.19	0.06	0.07	0.06	0.10	0.08	0.14	0.06	0.08
Distressed securities	0.00	0.03	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
Equity market neutral	0.06	0.00	0.16	0.09	0.16	0.17	0.13	0.15	0.12	0.11
Global macro	0.02	0.00	0.19	0.06	0.19	0.04	0.05	0.00	0.13	0.05
Long/short equity	0.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Merger arbitrage	0.45	0.00	0.19	0.32	0.19	0.00	0.27	0.00	0.27	0.29
Emerging markets	0.00	0.10	0.00	0.00	0.00	0.05	0.00	0.00	0.00	0.00
S&P500	−0.04	−0.72	−0.02	−0.05	−0.02	−0.03	−0.04	−0.10	−0.04	−0.05
Bond index	0.46	0.40	0.42	0.51	0.42	0.62	0.52	0.80	0.46	0.52
Large investors (representative strategy portfolios have 15 funds)										
Moments:										
Expected return	0.86	0.87	0.83	0.83	0.77	0.63	0.83	0.85	0.89	0.83
Standard deviation	0.65	1.59	1.37	0.72	1.38	0.83	0.67	1.55	1.23	0.67
Skewness	−0.48	1.12	1.06	0.25	0.99	0.12	0.02	1.12	0.88	0.01
Kurtosis	0.83	2.53	1.45	0.04	1.38	−0.71	−0.46	2.11	2.90	−0.44
Allocation:										
Convertible arbitrage	0.05	0.00	0.05	0.12	0.04	0.12	0.13	0.00	0.04	0.12
Distressed securities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity market neutral	0.16	0.00	0.00	0.00	0.00	0.16	0.00	0.00	0.00	0.00
Global macro	0.00	0.31	0.35	0.12	0.42	0.07	0.00	0.34	0.13	0.00
Long/short equity	0.00	0.39	0.25	0.00	0.15	0.00	0.00	0.35	0.30	0.00
Merger arbitrage	0.54	0.27	0.22	0.48	0.22	0.00	0.48	0.24	0.39	0.47
Emerging markets	0.00	0.00	0.00	0.00	0.01	0.05	0.00	0.00	0.00	0.00
S&P500	−0.07	−0.30	−0.21	−0.13	−0.19	0.02	−0.11	−0.27	−0.25	−0.10
Bond index	0.32	0.33	0.35	0.41	0.34	0.58	0.51	0.33	0.40	0.51

Table 8: **Moment statistics and optimal asset allocation across stocks, bonds, and hedge fund strategy classes with global constraints on capital investment.** Reported values are calculated from monthly net-of-all-fee returns. Investors assign capital to representative portfolios of convertible arbitrage, distressed securities, equity market neutral, global macro, long/short equity, merger arbitrage, emerging market hedge funds, the S&P 500 index, and the Salomon Brothers 7-Year Government Bond US Index. Optimal allocations are based on investors' preferences over expected return (α), skewness (β) and kurtosis (γ) using the PGP technique. The capital weight for each hedge fund strategy is constrained to be between zero and 30% and the capital weights for the stock and bond indices are both constrained to be within ±30%.

Portfolio	A	B	C	D	E	F	G	H	I	J
α	1.00	1.00	1.00	3.00	1.00	1.00	2.00	2.00	3.00	3.00
β	0.00	1.00	1.00	1.00	3.00	1.00	1.00	3.00	2.00	1.00
γ	0.00	0.00	0.25	0.25	0.25	0.75	0.75	0.25	0.25	0.50
Small investors (representative strategy portfolios have 5 funds)										
Moments:										
Expected return	0.81	0.75	0.78	0.77	0.73	0.71	0.77	0.76	0.77	0.76
Standard deviation	0.81	1.18	1.47	0.86	1.09	1.16	0.95	0.99	0.92	0.81
Skewness	−0.72	0.75	0.65	0.36	0.71	0.38	0.21	0.63	0.51	0.22
Kurtosis	1.11	1.84	0.61	0.45	1.21	−0.29	−0.31	1.18	0.98	0.05
Allocation:										
Convertible arbitrage	0.12	0.06	0.08	0.09	0.07	0.09	0.09	0.07	0.08	0.09
Distressed securities	0.05	0.00	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.00
Equity market neutral	0.19	0.15	0.14	0.19	0.21	0.30	0.24	0.18	0.17	0.23
Global macro	0.04	0.30	0.30	0.16	0.25	0.15	0.13	0.23	0.20	0.13
Long/short equity	0.00	0.00	0.24	0.00	0.00	0.21	0.13	0.00	0.00	0.00
Merger arbitrage	0.30	0.21	0.09	0.30	0.17	0.08	0.25	0.25	0.30	0.30
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S&P500	−0.01	−0.02	−0.15	−0.04	−0.01	−0.18	−0.13	−0.03	−0.05	−0.05
Bond index	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Large investors (representative strategy portfolios have 15 funds)										
Moments:										
Expected return	0.79	0.87	0.82	0.76	0.87	0.70	0.77	0.87	0.84	0.75
Standard deviation	0.63	1.41	1.21	0.67	1.41	0.88	0.71	1.41	1.18	0.60
Skewness	−0.32	1.05	0.97	0.44	1.05	0.47	0.16	1.05	0.95	0.04
Kurtosis	0.11	1.95	1.18	0.51	1.94	−0.38	−0.46	1.94	1.25	−0.35
Allocation:										
Convertible arbitrage	0.10	0.04	0.06	0.08	0.05	0.07	0.08	0.05	0.06	0.09
Distressed securities	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02
Equity market neutral	0.26	0.01	0.06	0.23	0.01	0.30	0.30	0.01	0.04	0.30
Global macro	0.01	0.30	0.30	0.14	0.30	0.15	0.09	0.30	0.29	0.04
Long/short equity	0.02	0.30	0.21	0.00	0.30	0.14	0.09	0.30	0.19	0.00
Merger arbitrage	0.30	0.30	0.26	0.30	0.30	0.14	0.30	0.30	0.30	0.30
Emerging markets	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00
S&P500	−0.03	−0.25	−0.19	−0.06	−0.26	−0.13	−0.11	−0.26	−0.19	−0.05
Bond index	0.30	0.30	0.30	0.30	0.30	0.30	0.25	0.30	0.30	0.30

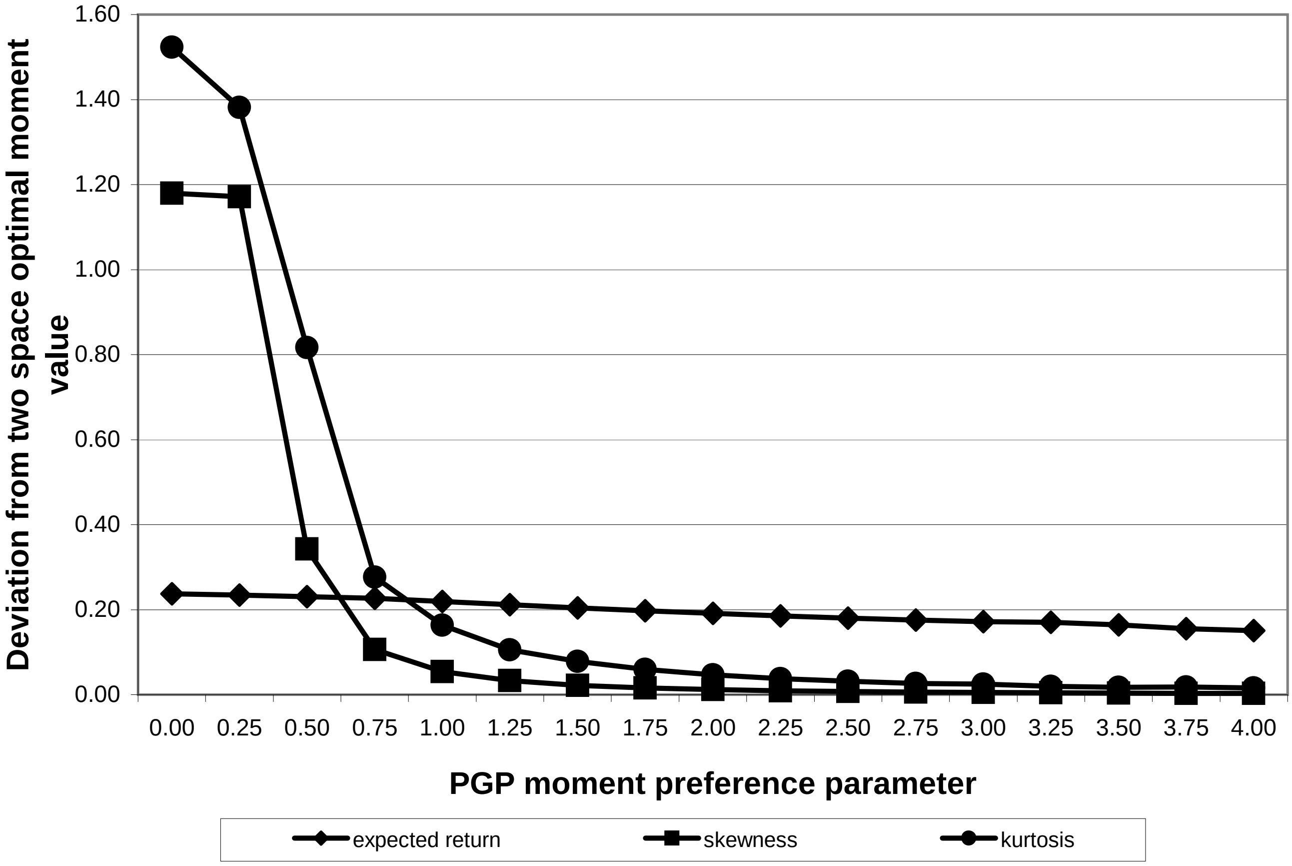

Figure 1: *This figure illustrates how the deviation from the two space optimal expected return varies as we vary the large investor's preference parameter over the given moment, holding the investor's preference over the other moments constant. For **expected return**, we hold* $\beta = 1$ *and* $\gamma = 0.5$ *constant and plot the deviation* d_1 *versus the preference parameter over expected return (*α*). The deviation is measured relative to* 0.97*, which is the highest obtainable expected return when variance is held constant at one. For **skewness**, we hold* $\alpha = 1$ *and* $\gamma = 0.5$ *constant, and plot the deviation* d_3 *versus the preference parameter over skewness (*β*). The deviation is measured relative to* 0.85*, which is the highest obtainable skewness value when variance is held constant at one. For **kurtosis**, we hold* $\alpha = 1$ *and* $\beta = 1$ *constant and plot the deviation* d_4 *versus the preference parameter over kurtosis (*γ*). The deviation is measured relative to* -0.10*, which is the lowest obtainable kurtosis value when variance is held constant at one.*

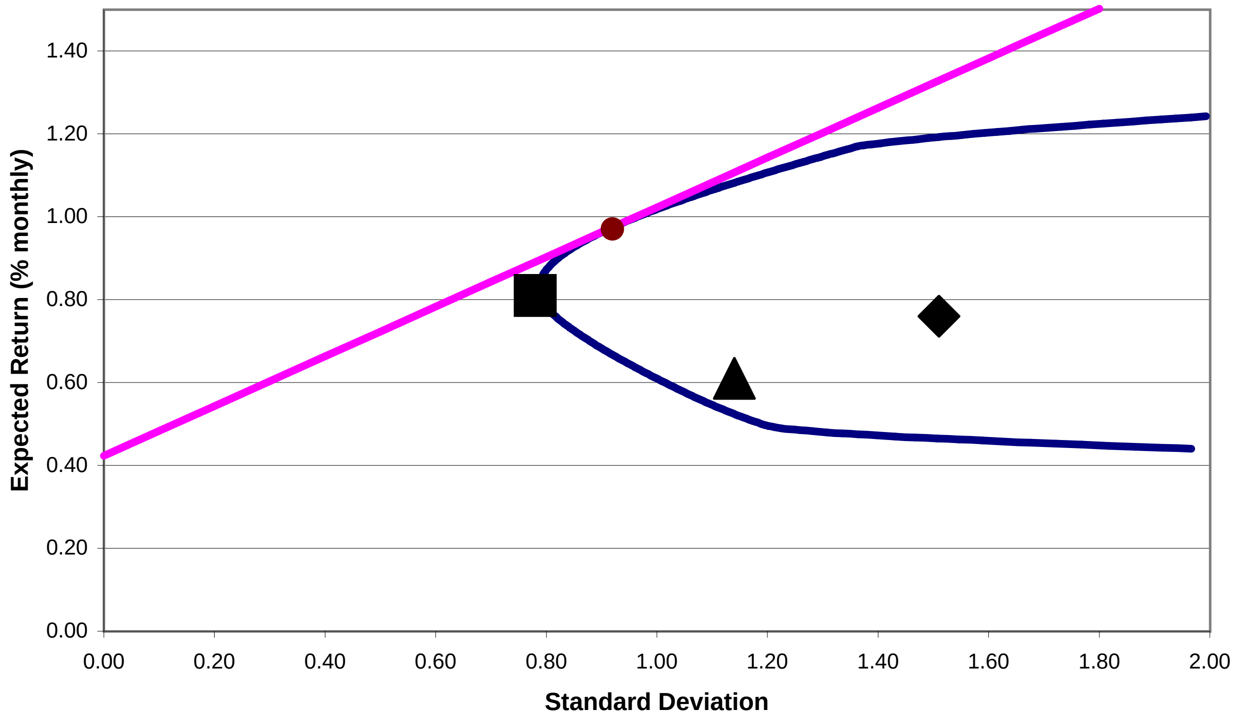

Figure 2: *This figure illustrates the feasible set of portfolios and the efficient frontier in a mean-variance framework for large investors. The square point indicates the optimal portfolio for* $(\alpha, \beta, \gamma) = (1, 0, 1)$. *The triangle point indicates the optimal portfolio for* $(\alpha, \beta, \gamma) = (0, 0, 1)$. *The diamond point indicates the optimal portfolio for* $(\alpha, \beta, \gamma) = (1, 1, 0)$.