9/24/2012 CLASS NOTES

- Class began referring back to 'Duration' definition slides

Closed Form Duration Equation PPT Slide:

DUR = N – (C/ P x R x [N – (1 + R)x 1/R [1 – 1 / (1+R)$^{\wedge N}$] (NOTE: This formula is in the powerpoint slides, but NOT the book!)

$$Dur = N - \left\{ \frac{INT}{(P_o \times r)} \times \left[N - ((1+r) \times PVIFA_{r,N}) \right] \right\}$$

= 4 – [90/ (1033.12) .08] [4 – 1.08 (3.31212684)]
= 3.56 years

P – the value of annuity

N = 4
i/y = 8
PMT = 90
FV = 1000

<u>EXAMPLE 1:</u> Suppose we have a 4 year bond with a 9% annual coupon, and an 8% EAR. Find the PV.

N = 4
PMT = 90
FV = 1000
i/y = 8
PV = 1033.121268

Duration: I------------------I-----------------I---------------I
90 90 90 1090

1[90/1.08] + 2[90/1.08$^{\wedge 2}$] + 3[90/1.08$^{\wedge 3}$] + 4[1090/1.08$^{\wedge 4}$]

9/1.08 + 180/1.08$^{\wedge 2}$ + 270/180$^{\wedge 3}$ + 4(1090)/1.08$^{\wedge 4}$ =

3656.719184/1033.121268 = 3.539486890

Now say the IR drops to 7%, how much will the bond fluctuate in PV?

ΔP = - D x P x ΔR/1+R = -3.53948689 x 1033.121268 x -.01/1.08

= 33. 858511 < bond will increase by this amount.

- Refer to 'Duration Example' Excel spreadsheet

Input sell amount at 2.

If one buys a 4 year semi-annual bond, and 2 years go by… what will it's PV be?

i/y = 8/2 = 4
PMT = 100/2 = 50
N = 4
PV = ????
FV = 1000

```
|--------|-------|--------|---------|---------|------|-------|
0       .5       1        2        2.5        3     3.5     4

                                    |--------|---------|-------|
                                    0       .5         1      1.5
```

PV is equal to 1036.29

Suppose the IR went up to 9%....

i/y = 9/2 = 4.5
PMT = 50
N = 4
FV = 1000
PV = ????

PV is equal to 1017.937628

EXAMPLE 2: Suppose we have an annual 4 year bond with a 9% coupon rate and a 8% EAR.

```
|------------------|-----------------|-----|----|-----|------|----------------|
0                  1                 2   2 ¼    2½ 2¾   3                4

              $90               90                     90          $1090
```

The bond is sold and bought once 2 ¼ years go by…
You can calculate this DIRECTLY using the regular rules of getting a PV by discounting each cash flow by 1+EAR raised to the number of years until you get that cash flow:

$90/(1.08^{.75}) + 1090/(1.08^{1.75}) = 1037.605621$

If on June 24, 2010 GM issued a 9% Annual coupon, lasting 4 years and with a i/y = 8%, one would want to purchase it today (9/24)! TODAY (9/24), this bond has 2.25 years remaining!

New trick: OK, we are standing at 2 ¼. We don't have a formula for that. So let's move up the timeline to TIME=3, where things look evenly spaced. Let's value the bond right at time period 3 where we basically just have a single payment of 90 right now and a bond that will pay us $1090 in one year.

```
I------------------I-----------------I-----I----I-----I------I-----------------I
0                  1                 2  2 ¼    2½ 2¾   3               4

              $90              90                  90         $1090
```

VALUE OF BOND at TIME=3 including the payment at 3:

1090/ (1.08) + $90 ←----------------$ 1099.25

OK, if this is the value at time period 3, what is the value at time period 2 ¼? Well, just discount that one payment back 3/4ths of a year:

1099.259259/ (1.08)^¾ = 1037.605621

NOTE: It is the same as before, when we did it directly. If we value everything to the period just after our "weird" period, then get that sum, and then bring that sum back to the "weird" period (i.e., in this case 2 ¼ years), we will get the correct answer

EXAMPLE 3: Suppose we have a bond that matures 21 ¼ years from now, it's a 7% annual bond with a 10% EAR.

```
I-------I-------I---------------------------------------------I---I--------I----I----------------I
0     ¼       1                                             19  ¼       20   1/4              21
```

Walk up 3 months mentally on the timeline… then we have a 21 year bond.

N = 21
PV = $740.5391713 + $70 = $810. 5391713 (three months from now)
…. Then bring this PV back to today! So = $791.4542887