Quick Problem Set #1

All rates are Effective Annual Rate (EAR) unless otherwise noted. Express all answers as an EAR

½ year rate:	2.5%	4 year rate:	5.25%
1 year rate:	3%	4.25 year rate:	5.50%
1.5 year rate:	3.5%	4.5 year rate:	5.6%
2 year rate:	4%	4.75 year rate:	5.75%
2.5 year rate:	4.25%	5 year rate:	6%
3 year rate:	4.5%	7 year rate:	7%
3.25 year rate:	4.75%	7.5 year rate:	7.5%
3.5 year rate:	4.85%	8 year rate:	8%
3.75 year rate:	5.00%	10 year rate:	10%

1. What is the one year forward rate between years 1 and 2?

   $$(1.03)^1(1+_1f_2)^1 = (1.04)^2 \qquad\qquad \rightarrow\; _1f_2 = 5.0097087\%$$

2. What is the one year forward rate between years 2 and 3?

   $$(1.04)^2(1+_2f_3)^1 = (1.045)^3$$

   $$(1+_2f_3)^1 = (1.045)^3/ (1.04)^2 \qquad\qquad \rightarrow\; _2f_3 = 5.5072231\%$$

3. What is the one year forward rate between ½ year and 1.5 years?

   $$(1.025)^{1/2}(1+_{0.5}f_{1.5})^1 = (1.035)^{1.5}$$

   $$(1+_{0.5}f_{1.5})^1 = (1.035)^{1.5}/(1.025)^{1/2} \qquad \rightarrow\; _{0.5}f_{1.5} = 4.0036526\%$$

4. What is the six month forward rate (expressed as an EAR) from ½ year to 1 year from now?

   $$(1.025)^{1/2}(1+_{0.5}f_1)^{1/2} = (1.03)^1$$

   $$(1+_{0.5}f_1)^{1/2} = (1.03)^1 / (1.025)^{1/2}$$

   $$(1+_{0.5}f_1) = [(1.03)^1 / (1.025)^{1/2}]^2 \qquad \rightarrow\; _{0.5}f_1 = 3.5024389$$

5. What is the three year forward rate starting two years from now?

   $$(1.04)^2(1+_2f_5)^3 = (1.06)^5$$

   $$(1+_2f_5)^3 = (1.06)^5 / (1.04)^2$$

   $$(1+_2f_5) = [(1.06)^5 / (1.04)^2]^{1/3} \qquad\qquad \rightarrow\; _2f_5 = 7.3546555\%$$

6. What is the one year forward rate starting three years and three months from now?

$$(1.0475)^{3.25}(1+_{3.25}f_{4.25})^1 = (1.055)^{4.25}$$

$$(1+_{3.25}f_{4.25})^1 = (1.055)^{4.25} / (1.0475)^{3.25} \quad \rightarrow \quad _{3.25}f_{4.25} = 7.9747857\%$$

7. What is the six month forward rate starting three years and three months from now?

$$(1.0475)^{3.25}(1+_{3.25}f_{3.75})^{0.5} = (1.05)^{3.75}$$

$$(1+_{3.25}f_{3.75})^{0.5} = (1.05)^{3.75} / (1.0475)^{3.25}$$

$$1+_{3.25}f_{3.75} = [(1.05)^{3.75} / (1.0475)^{3.25}]^2 \quad \rightarrow \quad _{3.25}f_{3.75} = 6.66396075\%$$

8. Under the unbiased expectations hypothesis, what do you expect the six month interest rate to be 3.75 years from now?

    NOTE: Unbiased Expectation hypothesis says expected rate = forward rate

$$E[_{3.75}r_{4.25}] = _{3.75}f_{4.25}$$

$$(1.05)^{3.75}(1+_{3.75}f_{4.25})^{0.5} = (1.055)^{4.25}$$

$$(1+_{3.75}f_{4.25})^{0.5} = (1.055)^{4.25} /(1.05)^{3.75}$$

$$1+_{3.75}f_{4.25} = [(1.055)^{4.25} /(1.05)^{3.75}]^2 \quad \rightarrow \quad _{3.75}f_{4.25} = 9.3266811\%$$

9. What is the two year forward rate starting eight years from now?

$$(1.08)^8(1+_8f_{10})^2 = (1.10)^{10}$$

$$(1+_8f_{10})^2 = (1.10)^{10} / (1.08)^8$$

$$1+_8f_{10} = [(1.10)^{10} / (1.08)^8]^{0.5} \quad \rightarrow \quad _8f_{10} = 18.377772928\%$$

10. What is the nine year forward rate starting one year from now?

$$(1.03)^1(1+_1f_{10})^9 = (1.10)^{10}$$

$$(1+_1f_{10})^9 = (1.10)^{10} / 1.03$$

$$1+_1f_{10} = [(1.10)^{10} / 1.03]^{1/9} \quad \rightarrow \quad _1f_{10} = 10.8065706\%$$

A. What is the EAR for 10% compounded semi-annually?

NOTE: Remember that "compounded semi-annually" is just CODE for "take the rate I just gave you and divide it in half and that is the rate you get over six months, i.e., the effective six-month (or effective semi-annual) rate"

So in this case, you get 5% every six months. So if you put a dollar in the bank, after six months it is $1.05. Now if you put that $1.05 in the bank for another six months, you get $1.1025

$(1.05)^2 = 1.1025$ so the Effective Annual Rate is 10.25%

B. What is the EAR for 10% compounded quarterly?

NOTE: Remember that "compounded quarterly" is just CODE for "take the rate I just gave you and divide it by four and that is the rate you get over three months, i.e., the effective three-month (or effective quarterly rate"

So in this case, you get 2.5% every three months. So if you put a dollar in the bank, after three months it is $1.025. Now if you put that $1.025 in the bank for another three months, you get $1.050625. Now if you put that in the bank for three months, you will earn 2.5% on that, and then do that one more time.

$(1.025)^4 = 1.103812891$ so the Effective Annual Rate is 10.3812891%

Quick note:

1. What is the effective semi-annual rate here?
    a. What does $(1.103812891)^{0.5}$ equal?
2. What does $(1.050625)^2$ equal?

C. You are going to get $100 in three months. Right now, your credit card is charging 12% compounded monthly. Assuming that is the correct rate, what is:
    a. The EAR on your credit card?

NOTE: Remember that "compounded monthly" is just CODE for "take the rate I just gave you and divide it by 12 and that is the rate you get over a months, i.e., the effective monthly rate"

$(1.01)^{12} = 1.126825030$ so the EAR is 12.6825030%

    b. The value today of that $100?
        i. $100 / (1.126825030)^{3/12}$ which is $100 / (1.126825030)^{0.25} = \$97.05901479$
        ii. $100 / (1.01)^3$ 97.05901479

Why does that work?