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SAMPLE PROBLEMS

1. Basic Time Value of Money – Price a Bond

1. What is the price of a six-year 5 % annual coupon bond with a face value of \$1000 if the discount rate is 6%, compounded monthly?

OK, the payment is \$50, the face value is \$1000, it is six years, and the interest rate is ... A HA! Since it is an annual bond, you need to get the EAY. 6% compounded monthly is $\frac{1}{2}\%$ per month, so $(1.005)^{12} = 1.061677812$, so you need to put 6.1677812% in for the interest rate.

Answer: \$942.8778779

2. Inflation and Real and Nominal

2. If the inflation rate is 2% and the real rate is 3%, what is the nominal rate? Assume all numbers are expressed as an EAY.

$$(1+\text{nominal}) = (1+\text{real}) \cdot (1+\text{inflation expectations})$$

Answer: 5.06%

3. Wall Street Journal

3. Describe what is going on with the London Stock Exchange.

4. Market Rates

4. What did the Dow Jones Industrial Average close at yesterday or today? You must be within 50 points.

DJIA close yesterday: _____

Or

DJIA close today: _____

5. Forward Rates

- A. Go to the last page and find the photocopy from the *Wall Street Journal*. Use the data on the last page to answer the questions below. Pretend that today is February 28th and that all of these bonds mature on the last day of the month. Also, assume that all yields or rates on the last page are expressed as an Effective Annual Rate (EAR).

Circle all bonds/notes/bills/issues/security that you use for the questions below. I need to be able to see which bond/note/bill/issue/security you used on the last page for which questions. Failure to circle the bonds will result in a loss of points.

1. Based on the information on the photocopied page, what do you expect the interest rate to be on a four-year bond issued three years from now? Express your answer as an EAR. (5 points)

6. Derive Duration

- B. Start with the price of a bond and use calculus to derive the duration formula.

7. Calculate Duration

- C. Assume that you own a four-year, 10% annual coupon bond with a face value of \$1000. The appropriate discount rate for this bond is 8% EAR.

1. What is the duration of the bond? (5 points)

Answer: 3.504213395

8. Use Duration to estimate how much a bond's price will change

4. Assume a bond has a price of \$950 and a duration of 7.65. If interest rates move from 7% EAR to 6% EAR, what is the new price of the bond based on its duration?

Answer: \$1017.920561

9. Calculate duration of a portfolio

5. What is the duration of a portfolio with \$100 in Bond A and \$300 in Bond B? Bond A's duration is 1.5; Bond B's duration is 4.

Answer: 3.375

10. Price of a Stock

6. What is the price of a stock where the next quarterly dividend is \$1.50, if dividends are expected to grow at 4% EAY and the appropriate discount rate is 12% EAY?

Answer: \$79.43258445

11. Limit Order Books

3. What is the difference between a limit order and a market order?

Answer: A market order specifies the stock, the direction (buy/sell), and then says buy right now at the market price, whatever it is. A limit order specifies the stock, the direction, and the price at which it can be executed, but not the time. As a result, it may never be executed.

Or, market order specifies time (immediately), but not price, while a limit order specifies price, but not when.

B. Here is some real data for IBM from the Archipelago limit order book:

IBM

ID	Price	Size	Time	ID	Price	Size	Time
ARCAEX	92.34	100	14:28:41	ARCAEX	92.38	1100	14:28:42
INET	92.34	600	14:28:40	INET	92.38	1300	14:28:42
ARCAEX	92.32	800	14:27:59	ARCAEX	92.40	1100	14:28:42
ARCAEX	92.31	1600	14:27:55	INET	92.40	1100	14:28:42
INET	92.31	1900	14:28:17	ARCAEX	92.41	1600	14:28:40
ARCAEX	92.20	400	14:27:55	INET	92.41	1600	14:28:40
ARCAEX	92.02	5000	10:59:16	ARCAEX	92.43	800	14:27:46
ARCAEX	91.78	2000	13:52:57	ARCAEX	92.61	400	09:43:15
INET	91.78	2000	13:52:57	ARCAEX	92.84	2000	14:27:37
ARCAEX	91.61	100	09:41:59	INET	92.84	2000	14:27:37

Answer the following questions independent of one another.

1. What is the current quote for IBM? Give both prices and amounts. (2 points)

BID PRICE/Depth: 92.34 / 700 shares ASK PRICE/Depth: 92.38 / 2400 shares

2. What is the cumulative depth on the ask side to 92.40? (3 points)

Cumulative Depth Answer: 4600 shares

3. If a limit order to buy 500 shares at 92.30 arrives, how does the limit order book change? What is the bid-ask spread after this new order arrives? Give both prices and amounts. (3 points)

Answer: The order goes onto the limit order book just under the 92.31. Now there is an order for 500 shares at 92.30 in the book. The bid ask does not change; it is still bid of 92.34 for 700 shares and an ask of 92.38 / 2400 shares.

12. Stock Markets

4. Explain the pertinent differences between the NYSE and Nasdaq.

Answer: The NYSE is an auction market that has a specialist, and a specific place (the floor) on which trades occur as well as the electronic submissions. Nasdaq has no specific place and is a competing dealer system. It does not have a specialist.

13. IPOs

5. Describe two primary differences between a preliminary prospectus and a final prospectus.

Answer: A preliminary prospectus has the red writing disclaimer on the side, no price or underwriting discount information on the front, no pricing information inside, and the amounts each underwriter will underwrite in the underwriting section is blank. The final prospectus does not have the red disclaimer, has the price of the deal and all underwriting information, including the underwriting discount on the front and the number of shares each underwriter has underwritten. Plus, it has the effective date of the IPO on it.

14. Calculate Payments on a Mortgage

1. Assume you want to buy a condo in Wellesley for \$1,000,000. You will have to put 20% down and finance the rest. Assume that you can borrow money at 7% (EAY) on a 15 year mortgage where the equal payments are made monthly, but you need to pay 2 points to get it.

A. How much of the 60th payment is for interest? (5 points)

OK, you are going to borrow \$800,000. You need a monthly interest rate $(1.07)^{1/12} = 1.005654145$, so your interest rate is : 0.5654145% per month. So, you are going to make 180 payments, with no final payment (FV=0), so

The payment is: \$7,094.797174

Now, you need the part of the 60th payment that is interest. To figure out the portion of a payment that is interest, you need to know the balance remaining after the payment just

before the payment in which you are interested. So, in this case, we need the balance that remains just after we make our 59th payment. The answer to that is the present value of all remaining payments after we make our 59th payment, i.e., the PV of the remaining 121 payments, which is \$620,507.5518.

Now, since we owed the bank \$620,507.5518 at the end of the 59th payment, one month later when we make our 60th payment, we must pay one month's interest on it. Since interest is 0.5654145% per month, we owe \$3,508.439671 in interest.

The answer is \$3,508.439671.

Note: The points are irrelevant for this question!

15. Calculate remaining balance on a mortgage

B. What is the remaining balance on the loan after you make the 60th payment above?

After you made your 60th payment, you have $180 - 60 = 120$ payments left. What is the present value of your remaining payments, at the interest rate and payment from A above?

Answer: \$616,921.1943

16. Covered Interest Parity

1. Look at the information from *The Wall Street Journal* attached to this exam. (Assume all numbers in the WSJ are expressed as an EAY.)

A. If you are going to borrow in the US and lend in Switzerland for three months, what do you need to charge in Switzerland on your loan if you want to make a 2% spread? (5 points)

Answer: Take whatever the US interest rate is. Assuming you can borrow at that rate, you need to make 2% more than that rate. You want to make 2% on it, so add 2% to it. (Let's assume that the US interest rate is 5%. This means that you want to make 7%.)

Then, just use covered interest parity:

$$\$1 * (1 + \text{us rate} + .02)^{1/4} = \$1 * \text{Spot rate SF/\$} * (1 + \text{rate in Switzerland})^{1/4} * (1 / \text{Forward rate in SF/\$})$$

17. Value of a Call option or a Put option

2. Describe the difference between options, futures, and forwards. (5 points)

Quick answer: Forwards are customized contracts between two parties where there is the right and the obligation to deliver a specified amount on a specified date. No money changes hands until that final date.

Futures also have the right and the obligation to trade, but are standardized contracts traded on a market. They are marked-to-market daily.

Options provide the right but not the obligation to trade. You have to pay a premium to get an option when you buy the option.

18. Calculate the number of futures needed to immunize a bank from changes in interest rates.

2. Currently, a bank has \$50,000,000 in cash and one bond as assets, a three year 8% annual coupon bond with a face value of \$150,000,000. The bank also has liabilities currently worth \$125,000,000. Assume that the current interest rate on all bonds/loans/liabilities is 10%, and that the duration of the liabilities is one year.

A. If interest rates rise by 1%, by how much will the equity of the bank change? (10 points)

This is a big one to answer. First, what is the value of the bond? $N=3$, $PMT=.08*\$150$ million = \$12 million is the payment, $FV=\$150$ million, and the interest rate is 10%, so the value of the bond is \$142,539,444. So, total assets must be \$50 million more, or \$192,539,444.

OK, what is the duration of the bond? Well, calculate it out. If you do, you should get 2.777356104. Now, the duration of the assets is the weighted average of the duration of the assets, weighted by their market value. The duration of the assets is then:

$$\text{Duration of assets} = D_{\text{cash}} * (50,000,000/192,539,444) + D_{\text{bond}} * (142,539,444/192,539,444)$$

Since the duration of cash is zero, this falls to:

$$\text{Duration of assets} = 2.777356104 * (142,539,444/192,539,444) = 2.056112694$$

So, now put into the formula

$$\Delta E = -(D_A - kD_L) * A * \frac{\Delta R}{1 + R}$$

So, the change in equity =

$$\begin{aligned} & -(2.056112694 - (125,000,000/192,539,444)*1) * 192,539,444 * (.01/1.1) \\ & = -\$2,462,570.863 \end{aligned}$$

B. Assume that there exists a six-month treasury bond future selling for \$96,000 where the underlying treasury bond is a 10-year 7% annual coupon bond with a face value of \$1000 which has a duration of 6 years. How many futures do you need to either

buy or sell to hedge the interest rate exposure of the bank? (5 points)

Put into the formula:

$$N_F = \frac{-(D_A - kD_L) * A}{P_F * D_F}$$

$$\begin{aligned} \text{So, } N_F &= -[(2.056112694 - (125,000,000/192,539,444)*1) * 192,539,444] / (96,000 * 6) \\ &= -470.28 \text{ futures, or short 470 futures.} \end{aligned}$$

19. Understand mutual funds

3. Explain how to calculate Net Asset Value