

LEVERING AND UNLEVERING BETAS

Purpose. To show the relation between a company's asset, or unlevered, beta and its equity beta, and to demonstrate why one might want to know this relation.

Derivation. By definition, the market value of a levered firm equals the market value of its debt plus the market value of equity.

Modigliani and Miller tell us that the value of a levered firm can be written as the value of the firm unlevered plus the present value of the tax shields due to debt financing. (This relationship assumes perpetual debt and ignores any bankruptcy or distress costs accompanying debt financing.)

Equating these two expressions,

$$D + E = V_u + tD$$

where D is interest-bearing debt, E is the market value of equity, V_u is the value of the firm without any debt, and t is the marginal tax rate.

An important property of beta is that the beta of a portfolio is the weighted-average of the betas of the individual assets comprising the portfolio. Applying this insight to both sides of the equation above,

$$\frac{D}{D + E} \beta_D + \frac{E}{D + E} \beta_E = \frac{V_u}{V_u + tD} \beta_u + \frac{tD}{V_u + tD} \beta_{ITS}$$

where β_D is the beta of debt, β_e is the beta of equity, β_u is the beta of the unlevered firm, or equivalently, the firm's asset beta, and β_{ITS} is the beta of the firm's interest tax shields.

Now two assumptions: 1) The firm's debt is for practical purposes riskless, so that β_D equals 0, and 2) the risk of interest tax shields equals the risk of the firm's unlevered asset cash flows, so that $\beta_{ITS} = \beta_u$.¹

Given these assumptions, the above equation simplifies to

$$\beta_u = \frac{E}{D + E} \beta_E$$

In words, a firm's asset beta equals its equity-to-value ratio times its equity beta.

An Example. To illustrate the use of this equation, suppose Zenos Manufacturing is contemplating investing in a new business. An existing company in the target industry has an equity beta of 0.90 but is very conservatively financed with a market value, equity-to-value ratio 95%. Zenos intends to use an equity-to-value ratio of 30% and wonders what beta to use in estimating a cost of capital in the new business.

¹ A popular alternative assumption is that $\beta_{ITS} = \beta_D = 0$, which yields the following relation between asset beta and equity beta,

$$\beta_u = \left[\frac{1}{1 + (1-t)\frac{D}{E}} \right] \beta_E$$

This alternative assumption may be plausible for existing debt, but if firm indebtedness varies over time with firm value, it is much less plausible for new debt. For new debt I think the more reasonable assumption is that $\beta_{ITS} = \beta_u$ as above. I prefer this assumption because it is at least as plausible as the alternative, it yields a simpler expression, and it is consistent with traditional WACC valuation.

Zenos needs to unlever and then re-lever the target firm's equity beta. Unlevering the target's equity beta yields an estimated asset beta for the new industry. Thus, $\beta_u = 0.95 \times 0.90 = 0.855$. Relevering this asset beta to reflect Zenos target capital structure yields the appropriate equity beta for Zenos to use in

estimating a cost of capital in the new business. $\beta_E = \frac{D+E}{E} \beta_u = \frac{1}{0.30} 0.855 = 2.85$.